Effect of Spatial, Temporal, and Velocity Resolution on Doppler Tissue Image Derived Indices

JL Rojo-Álvarez¹, J Bermejo², AB Rodrígez-González¹, R Yotti², A Martínez-Fernández¹, MA García-Fernández², JC Antoranz³

¹Universidad Carlos III de Madrid, Leganés, Madrid, Spain
²Hospital General Universitario Gregorio Marañón, Madrid, Spain
³Universidad Nacional de Educación a Distancia, Madrid, Spain

Abstract

Many indices derived from color Doppler Tissue Imaging (DTI) are computed mathematically from the velocity fields provided by the ultrasound scanner. We developed a procedure for assessing the uncertainty that temporal, spatial, and velocity resolution causes on cardiovascular parameters derived from myocardial velocity. A first-order Taylor’s series for the functional relationship between velocity and the cardiac index under study is proposed, and analytically solved by using the chain rule. Performance on myocardial acceleration, strain, and strain rate, was tested both on a simulation model (symbolic calculations) and on a clinical image example. Velocity and temporal resolutions were the most determinant on DTI-derived parameters, and increasing color-Doppler bit-depth from 5 to 7 bits reduced to less than 10% the uncertainty of all the analyzed indices. Taylor-based error bounds are a useful tool to assess the effect of resolution on quantitative cardiovascular indices derived from tissue velocity measurements.

1. Introduction

Doppler echocardiography has become the most useful noninvasive technique to assess cardiovascular function. Pulsed-wave and color-Doppler Tissue Imaging (DTI) modalities were developed during the last decade [1, 2]. Based on the modification of Doppler flow filters, DTI provides accurate measurements of the velocity of the myocardial walls.

Most useful cardiac indices derived from DTI raw velocity data require actually mathematical operations. Endocardial tracking [3], myocardial acceleration [4], myocardial strain and myocardial strain-rate [5, 6] have demonstrated to be extremely sensitive for detecting and quantifying subtle cardiovascular abnormalities previously undetectable non-invasively. Mathematical derivation of color-Doppler velocity-data is strongly dependent on noise and artifacts [7], and indices numerically derived from these images are heavily influenced by the lack of signal ("black holes"). Scanner resolutions in time (sampling frequency), space (pixels/cm), and velocity (bit-depth of the color-Doppler processor), are additional sources of uncertainty. But surprisingly, the effect of image resolution on indices derived from measurements of velocity has not been yet specifically addressed. Our aim is to propose a general method suitable to determine the magnitude of error that the resolution of the image acquisition system induces on each cardiac index. For this purpose, an analytical framework is proposed, from which the maximal error related to current ultrasound scanner resolutions can be calculated, in order to define where further technological efforts should focus on.

The draw of the paper is as follows. In the next section, error bounds for myocardial acceleration, strain, and strain-rate, are derived analytically, based on a first-order Taylor’s series. The simulation model used is then described. Simulations of tissue deformation during a cardiac cycle are used to measure the impact of spatial, temporal, and velocity resolutions on DTI indices. Error bounds are provided for a true example of DTI image. Finally, conclusions are summarized.

2. Error bounds

Myocardial acceleration, strain, and strain-rate, can be expressed through mathematical operations that transform the velocity fields. A simple first order Taylor’s series expansion can approximate this operator by a linear expression, and then, how each simple operation involved affects the cardiac index can be assessed from the chain rule. This sensitivity analysis procedure has been widely used to study the effect of coefficient quantization in structures for digital filters [8].
**Error bounds using Taylor’s series.** Be \( v_i(s, t) \) a tissue 1D+\( t \) velocity field (M-mode) that is measured as a function of time \( t \) and distance \( s \) from the transducer. Be \( f \) a cardiac index that is obtained by functional operations on \( v_i(s, t) \). By taking the first order Taylor’s series expansion of \( f \) in a given point \((s, t, v_i(s, t))\), and for small increments of time \( (\delta t) \), space \( (\delta s) \), and velocity \( (\delta v_i) \), we can express changes \( \delta f \) as

\[
\delta f \simeq \frac{\partial f}{\partial s} \cdot \delta s + \frac{\partial f}{\partial t} \cdot \delta t + \frac{\partial f}{\partial v_i} \cdot \delta v_i \tag{1}
\]

This equation represents an approximation to the influence that small increments on velocity, space, and time, produces on the increments of cardiac index \( f \). Given that velocity is expressed as a function of time and space, coefficients of \( \delta s, \delta t, \delta v_i \), are indeed two-variable expressions.

Without loss of generality, we will use a temporal cardiac index defined by a (continuous and differentiable) time function \( f = f(t) \) to analyze time-varying waveforms. To obtain an upper bound of the effect of each resolution, it is assumed that the total influence of each partial derivative in the right term of (1) is mainly due to its value at instant \( t \). For a spatial domain \( S \), we have

\[
|\delta f(t)| \leq c_s(t)\delta s + c_t(t)\delta t + c_v(t)\delta v_i \tag{2}
\]

where

\[
c_v(t) = \int_S \left| \frac{\partial f(t)}{\partial v_i(s, t)} \right| ds \tag{3}
\]

and \( c_s(t), c_t(t) \) can be calculated in a similar way.

The main issue in this procedure is the calculation of \( \frac{\partial f}{\partial v_i} \), the other partial derivatives being obtained directly by the chain rule as:

\[
\frac{\partial f}{\partial s} = \frac{\partial f}{\partial v_i} \frac{\partial v_i}{\partial s} \tag{4}
\]

\[
\frac{\partial f}{\partial t} = \frac{\partial f}{\partial v_i} \frac{\partial v_i}{\partial t} \tag{5}
\]

Spatial domain of integration \( S \) is just the straight line along the spatial dimension of the image. However, if this domain integration results in too relaxed error bounds, \( S \) can be conveniently reduced according to previous a priori knowledge of the operator and/or the image, as it will be shown in the applications. Note that (2) is a highly pessimistic bound, because all errors are assumed to take place in the same orientation.

**DTI derived indices.** Myocardial acceleration is a measurement of the changes in tissue velocity along the ultrasound streamline, and it is computed \[4\] as

\[
a(s, t) = \frac{\partial v_i(s, t)}{\partial t} \tag{6}
\]

Natural strain-rate accounts for the speed at which deformation of an object occurs, and, in the case of cardiac strain rate, it represents the rate at which a given location of the myocardial wall thickens and thins. It can be obtained using DTI \[9\] as

\[
\ddot{\varepsilon}(s, t) = \frac{v_i(s, t) - v_i(s - L, t)}{L} \tag{7}
\]

where \( \ddot{\varepsilon}(s, t) \) is the relative amount of cardiac wall deformation between two locations separated by a distance \( L \), usually \( L = 1 \text{ cm} \). Strain accounts for the deformation of an object, normalized to its original shape. The total amount of cardiac strain can be obtained from DTI \[9\] by simply adding infinitesimal strain contributions,

\[
\varepsilon(s, t) = \int_{t_0}^t \ddot{\varepsilon}(s, \tau) d\tau \tag{8}
\]

In M-mode, acceleration, strain rate, and strain of the myocardium are also 1D + \( t \) fields (i.e., images). For simplicity, each cardiac index is obtained at a single spatial location \( s_0 \), providing a waveform of its temporal variation.

Partial derivatives of (6), (7), (8) with respect to \( v_i(s, t) \) are given by:

\[
\frac{\partial a(s, t)}{\partial v_i(s, t)} = \frac{\partial v_i(s, t)}{\partial t} \tag{9}
\]

\[
\frac{\partial \dot{\varepsilon}(s, t)}{\partial v_i(s, t)} = \frac{1}{L} \left( 1 - \frac{\partial v_i(s - L, t)}{\partial t} \left( \frac{\partial v_i(s, t)}{\partial t} \right)^{-1} \right) \tag{10}
\]

\[
\frac{\partial \varepsilon(s, t)}{\partial v_i(s, t)} = \int_{t_0}^t \frac{\partial \dot{\varepsilon}(s, \tau)}{\partial v_i(s, \tau)} d\tau \tag{11}
\]

### 3. Image model

A simple model of DTI during a full cardiac cycle was created from 6 individual components, each of which combined an exponential and a polynomial expression, as

\[
v_i(s, t) = \left( \frac{8}{4} - 1 \right) \sum_{i=1}^{6} a_i \exp \left\{ -\left( \frac{t - t_i}{\tau_a} \right)^2 \right\} \tag{12}
\]

where \( t_a = 0.05 \text{ s} \). The six gaussian components correspond to diastolic early filling \( (i = 1, 2) \), atrial contraction \( (i = 3) \), isovolumic contraction \( (i = 4) \), and (fast and slow) systolic contraction \( (i = 5, 6) \) (Fig. 1). Time 0 was defined at QRS onset. Constant parameter values \( a_i \) and \( t_i \) were adjusted based on the following constrains:

- Physiological values for myocardial velocity (systolic and diastolic E-wave and A-wave), acceleration, strain, and strain-rate \[4, 5\].
4. Results

Simulations. Discrete DTI images were simulated from the myocardial velocity model using the following two sets of resolutions:

- typical values of most ultrasound scanners, given by
  \[
  \delta t = \frac{1}{200} \text{s}, \quad \delta s = \frac{1}{20} \text{cm/pixel}, \quad \text{and} \quad \delta v_t = \frac{\text{upper - lower Nyquist limits}}{2^b} \quad \text{cm/s},
  \]
  where \( b \) is the number of bits (eg, for color scale limits of \( \pm 10 \text{ cm/s} \) and \( b = 5 \) bits, \( \delta v_t = 20/32 = 0.625 \text{ cm/s} \) [10, 11];

- improved resolution values, identified as the temporal, spatial, and velocity resolution values that reduced the maximum of the error bound below 10% of the peak of the cardiac index.

Acceleration, strain, and strain rate waveforms were calculated after a \( 3 \times 3 \) median filtering, by discrete numerical derivation and/or integration of the velocity image.

Figure 2(a) shows the theoretical error bounds calculated analytically for scanner resolutions. Spatial resolution is appropriate for the three DTI indices. However, errors due to time and velocity resolutions are relevant for all of them. Velocity resolution is specially important to accurately measure peak myocardial acceleration values. Spikes in strain rate bounds of velocity are observed because (10) needs to be regularized by adding a small constant (0.1) to the denominator, in order to avoid dividing by zero. Noteworthy, strain error due to

<table>
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<th>3</th>
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Table 1. Constant parameter values used in the DTI model defined in (12).

Figure 1. Waveforms from simulated color DTI M-mode, measured at \( s_0 = 9 \text{ cm} \): (a) velocity, (b) myocardial acceleration, (c) strain rate, and (d) strain.

Figure 2. DTI error bounds obtained from the simulated myocardial velocity models, for nominal values of current ultrasound scanners. Error bounds for myocardial acceleration (up), strain rate (middle) and strain (down).

Figure 3. Example of resolution errors estimated from a clinical DTI image. (a) Color M-mode DTI of the mid-basal interventricular septum from a healthy volunteer; (b) Numerically estimated strain (up) and estimated Taylor error bounds for calculating total strain (down).
velocity clearly increases over time, as a consequence of the integral operator involved in its calculation.

Table 2 presents the theoretical values required to decrease errors to 10% of the peak amplitude of each index. Velocity resolution for accurate acceleration measurements seems to be the most expensive (almost 7 bits per sample). An increase in sampling frequency is convenient for both acceleration and strain.

Application example. The image from a healthy volunteer is shown in Fig. 3(a). Scanner resolutions were \( \delta_s = 1/20 \text{ cm/pixel}, \ \delta_t = 1/200 \text{ s}, \) and 5 bits (equivalent to 1.25 cm/s). Error bounds were obtained for the raw image, with the same preprocessing as in previously modeled images. For the three indices, all the resolutions were shown to be poor. As an example, Fig. 3(b) presents the estimated error bounds for the estimated strain waveform. A typical error of current methods used for calculating myocardial strain from DTI curves can be observed: it does not return to baseline at the end of a cardiac cycle. In this example, velocity appears as the most important source of error.

5. Conclusions

A method for quantifying the impact of spatial, temporal, and velocity resolution on cardiac indices derived from DTI has been proposed, basing on a first order Taylor’s series expansion of the functional that transforms the tissue velocity field into the numerical index. The efficacy of the method has been tested on a completely known simulation model. Though the error will be certainly dependent on the acquired image, the method can be applied to true images, still providing with the order of magnitude of the error. The statement of bounds for indices derived from color-Doppler flow images, as well as the use of the method in extensive image data bases, are the most immediate future applications.

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