Space Rescaling in the Method of Fundamental Solution Improves the ECGI Reconstruction

Pauline Migerditichan[1 2 3], Mark Potse[1 2 3], Nejib Zemzemi[1 2 3]

[1] Institute of Mathematics, University of Bordeaux, Talence, France
[2] INRIA Bordeaux Sud-Ouest, CARMEN Team, Talence, France
[3] IHU Liryc, Electrophysiology and Heart Modeling Institute, Pessac, France

Abstract

The method of fundamental solutions (MFS) has been extensively used for the electrocardiographic imaging (ECGI) inverse problem. One of its advantages is that it is a meshless method. We remarked that the using cm instead of mm as a space unit has a high impact on the reconstructed inverse solution. Our purpose is to refine this observation, by introducing a rescaling coefficient in space and study its effect on the MFS inverse solution. Results are provided using simulated test data prepared using a reaction-diffusion model. We then computed the ECGI inverse solution for rescaling coefficient values varying from 1 to 100, and computed the relative error (RE) and correlation coefficient (CC). This approach improved the RE and CC by at least 10 % but can go up to 40 % independently of the pacing site. We concluded that the optimal coefficient depends on the heterogeneity and anisotropy of the torso and does not depend on the stimulation site. This suggests that it is related to an optimal equivalent conductivity estimation in the torso domain.

1. Introduction

Electrocardiographic imaging is a noninvasive technique to assess the electrical potential on the epicardial surface from measurements realized on the torso surface. A meshless approach that employs the MFS technique has been adopted [1] to solve the inverse problem of electrocardiography. Its performance is similar to other, more elaborate techniques such as finite-element models [2]. However, the MFS assumes a homogeneous isotropic torso conductivity. In reality several organs have high deviating conductivities and some, such as the skeletal muscle, are strongly anisotropic. These heterogeneities have an important effect on forward solutions of the surface potentials [3, 4], and therefore might affect the quality of the MFS solution as well. To assess the severity of this problem we tested the MFS on data generated with homogeneous, inhomogeneous, and inhomogeneous-anisotropic forward models. Then, we investigated whether a scaling factor added to the MFS kernel can compensate for it.

2. Methods

2.1. Standard MFS

The solution given by the MFS approach is represented in the form of a linear superposition of source functions located on a set of virtual source points $y_j, j = 1 \dots M$ over an auxiliary surface [1]. The method uses a set of measured body surface potentials $u(x_i)$ at electrode positions x_i, $(x_i | i = 1 \dots N)$ and attempts to express these in terms of the source functions with weight factors a_j. The source functions and a_j can then be used to predict the potential at any point on the torso or cardiac surface.

When using Dirichlet and Neumann conditions we obtain the linear system

$$A\vec{a} = \vec{b} \tag{1}$$

where

$$\begin{aligned} \vec{a} &= (a_0, a_1, \dots, a_M)^T, \\ \vec{b} &= (u(x_1), \dots u(x_N), 0, \dots, 0)^T, \end{aligned}$$

and

$$A = \begin{pmatrix} 1 & f(\|x_1 - y_1\|) & \cdots & f(\|x_1 - y_M\|) \\ \vdots & \vdots & \cdots & \vdots \\ 1 & f(\|x_N - y_1\|) & \cdots & f(\|x_N - y_M\|) \\ 0 & \dfrac{\partial f(\|x_1 - y_1\|)}{\partial n} & \cdots & \dfrac{\partial f(\|x_1 - y_M\|)}{\partial n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \dfrac{\partial f(\|x_N - y_1\|)}{\partial n} & \cdots & \dfrac{\partial f(\|x_N - y_M\|)}{\partial n} \end{pmatrix} \tag{2}$$

 ISSN: 2325-887X DOI: 10.22489/CinC.2019.333

in which $f(r) = 1/(4\pi r)$ is the fundamental solution of Laplace's equation in 3D, and $\|x - y\|$ is the 3D euclidean distance between points x and y.

2.2. MFS with scaling factor

We added a rescaling coefficient α to the kernel of the MFS matrix. The approximate solution of the MFS relies upon the Laplace fundamental solution which hinges upon the euclidean distance $r = \|x - y\|$. We made the assumption that reducing the distance will help to make our problem better conditioned. To do this let R be the new reduced distance expressed by:

$$R = \frac{r}{\alpha} \quad \text{, with} \quad \alpha \geq 1$$
$$f(R) = f(\frac{r}{\alpha}) = \alpha f(r). \tag{3}$$

Similarly,

$$\frac{\partial f(R)}{\partial n} = \frac{\partial f(\frac{r}{\alpha})}{\partial n} = \alpha^2 \frac{\partial f(r)}{\partial n} \tag{4}$$

Taking this new distance R in the MFS matrix (2) amounts to use the old distance r in the matrix A_α given by the expression

$$\begin{pmatrix} 1 & \alpha f(\|x_1 - y_1\|) & \cdots & \alpha f(\|x_1 - y_M\|) \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \alpha f(\|x_N - y_1\|) & \cdots & \alpha f(\|x_N - y_M\|) \\ 0 & \dfrac{\alpha^2 \partial f(\|x_1 - y_1\|)}{\partial n} & \cdots & \dfrac{\alpha^2 \partial f(\|x_1 - y_M\|)}{\partial n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \dfrac{\alpha^2 \partial f(\|x_N - y_1\|)}{\partial n} & \cdots & \dfrac{\alpha^2 \partial f(\|x_N - y_M\|)}{\partial n} \end{pmatrix} \tag{5}$$

In both cases the Tikhonov regularization method [5] with a fixed parameter $\lambda = 10^{-2}$ was used to stabilize the solution and obtain $\vec{a}$.

2.3. Simulated data

Test data were prepared with a reaction-diffusion model of the heart on a finite-difference mesh with 0.2 mm resolution. Computed transmembrane currents were transferred to a torso model with 1-mm resolution to compute torso potentials [6]. Three sets of parameters were used for the torso conductivities. The first was fully homogeneous and isotropic (HOM), the second piecewise heterogeneous (lungs, liver, blood, skeletal muscles, and the remaining tissue) (HET) and isotropic, and the third model was the same as the second but the skeletal muscles were anisotropic (ANISO). We simulated three cases with different stimulation sites: mid anterior septal junction (MASJ), high anterior septal junction (HASJ) and left ventricular septum (LVS). All simulations were performed with the Propag-5 software [7] on a Bullx cluster machine.

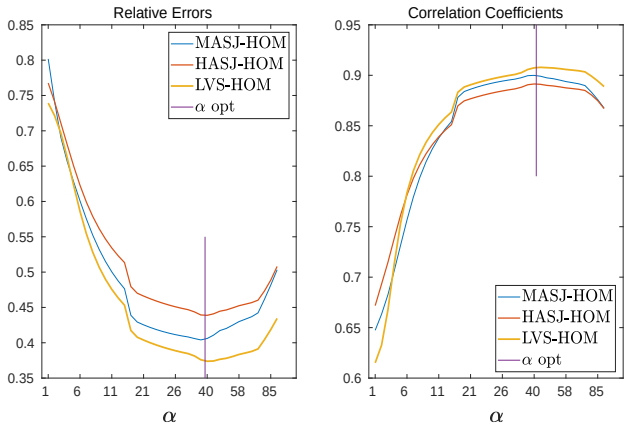


Figure 1. Relative errors and Correlation coefficients of homogeneous model (HOM) with pacing site in mid anterior septal junction (MASJ), high anterior septal junction (HASJ) and left ventricular septum (LVS).

2.4. Evaluation of reconstructed potentials

To compare the reconstructed potentials on the epicardium with the simulated ones correlation coefficients (CC) and relatives errors (RE) were computed through time and then averaged to get only two values for each scaling factor. RE quantifies the amplitude difference and CC the pattern similarity between the simulated and the computed potentials.

3. Results

We studied the influence of the α factor inside the MFS matrix for $\alpha \in [1, \ldots, 95]$ with three different pacing sites in each of the three models. Results for a completely homogeneous and isotropic torso model are shown in Figure 1, those for a heterogeneous and isotropic one in Figure 2, and those for a heterogeneous model with anisotropic skeletal muscle in Figure 3.

The results show that RE and CC vary widely with α and that the shape of the error function was different for each model. For the HOM model, small values of α resulted in the largest errors (Figure 1), whereas for the ANISO model they resulted in the smallest errors (Figure 3). For the HET model, both small and large α values resulted in

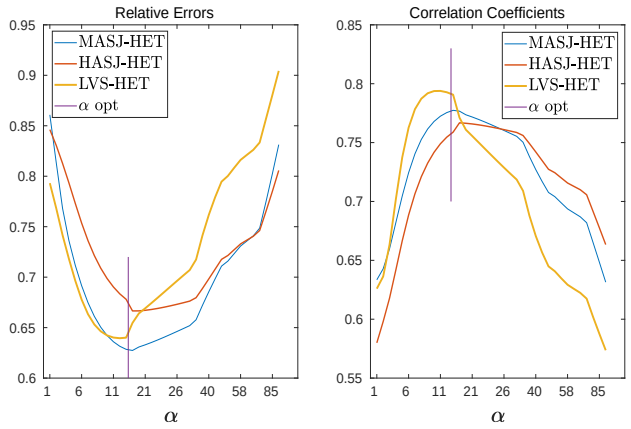


Figure 2. Relative errors and Correlation coefficients of heterogeneous and isotropic model (HET) with pacing site in mid anterior septal junction (MASJ), high anterior septal junction (HASJ) and left ventricular septum (LVS).

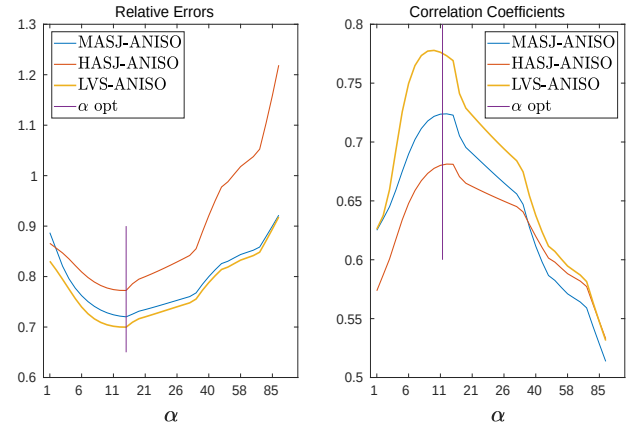


Figure 3. Relative errors and Correlation coefficients of heterogeneous and isotropic with anisotropic skeletal muscles model (ANISO) with pacing site in mid anterior septal junction (MASJ), high anterior septal junction (HASJ) and left ventricular septum(LVS).

relatively large errors and the optimum solution was found for intermediate values.

For all six cases an "optimal" scale factor could be found. The effect of this factor on the reconstruction of the signal is summarized in tables 1 and 2.

To illustrate the impact of α-optimization on reconstructed electrograms we reconstructed the signal on the heart for the standard MFS approach and the one with optimal scaling and compared them to the simulated one for a location near the stimulation site (HASJ in this case) as shown in Figure 4.

Table 3 shows the estimated activation time at the pacing site position. It is defined as the instant of steepest

Pacing Site	Model	MFS	Scaled MFS
MASJ	HOM	0.80	0.40
	HET	0.86	0.63
	ANISO	0.88	0.72
HASJ	HOM	0.77	0.45
	HET	0.85	0.67
	ANISO	0.87	0.77
LVS	HOM	0.74	0.37
	HET	0.83	0.69
	ANISO	0.83	0.64

Table 1. Mean RE of the reconstructed potentials along the time for the standard MFS Method and the "optimal" scaled method.

Pacing Site	Model	MFS	Scaled MFS
MASJ	HOM	0.65	0.90
	HET	0.63	0.78
	ANISO	0.62	0.72
HASJ	HOM	0.67	0.89
	HET	0.58	0.77
	ANISO	0.57	0.68
LVS	HOM	0.61	0.91
	HET	0.63	0.79
	ANISO	0.63	0.78

Table 2. Mean CC of the reconstructed potentials along the time for the standard MFS Method and the "optimal" scaled method.

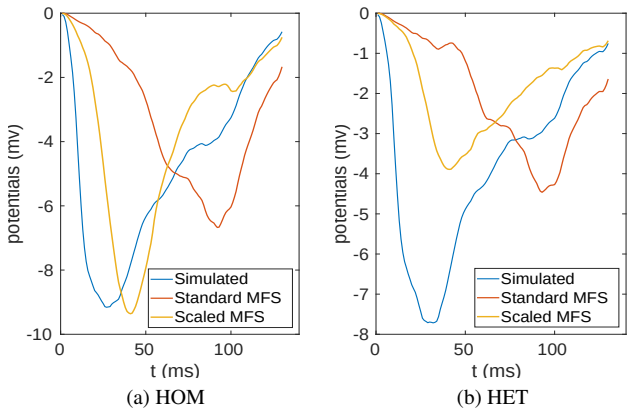
Model	Simulated	MFS	Scaled MFS
HASJ-HOM	9 ms	54 ms	25 ms
HASJ-HET	9 ms	55 ms	23 ms
HASJ-ANISO	9 ms	52 ms	22 ms

Table 3. Estimated activation time near the pacing site for the standard MFS Method and the "optimal" scaled method.

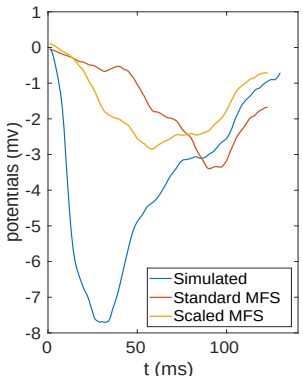
downstroke in the electrogram. The error approximates 45 ms for the original MFS method in each case, and approximately 15 ms error for the scaled MFS approach. The modified method was also better at reconstructing the deep S wave seen in the simulated signal which explains the improvements in the activation time reconstruction.

4. Discussion and conclusions

By applying the MFS to torso potentials with homogeneous and inhomogeneous torso models we have shown that torso heterogeneity reduces the quality of the MFS inverse solution, both in terms of RE and CC. In addition we have shown that the use of a scaling factor in the MFS can reduce the RE in estimated cardiac potential by upto



(a) HOM (b) HET



(c) ANISO

Figure 4. Comparison of reconstructed potentials for HASJ - standard MFS (red line) and scaled MFS (orange line) - against simulated one (blue line) in a point near the pacing site for three different torso models (a), (b) and (c).

50 % and improve the CC by upto 30 percent-points. We conclude that the optimal coefficient depends on the heterogeneity of the torso and does not depend on the stimulation site. In fact for the three stimulation cases the value of the optimal coefficient was 40 (respectively, 13 and 18) for the HOM (respectively HET and ANISO) case. We can suppose that the obtained optimal coefficients are correlated with an optimal equivalent conductivity in the torso domain.

In terms of RE and CC our correction worked best in case of a homogeneous torso model. Thus, even with this correction, the torso heterogeneity remains a problem for the MFS. Yet, if we focus on activation time this method shows some promising results by reducing errors by around 30 ms. This is explained by the fact that the slope of the electrograms obtained with the rescaled method is better captured than with the standard one.

In this study we fixed the Tikhonov factor to understand the results more easily. Using a method to find the optimal regularization factor as described by Barnes and Johnston [8] could improve them. Moreover, to determine the optimal value of α we need to know the true solution. For practical applications it would be of interest to develop a method looking for both the optimal regularization factor and the optimal scaling factor at the same time.

Acknowledgments

This work was granted access to HPC resources of CINES under GENCI allocation 2019-A0050307379. This work was also supported by the French National Research Agency, grant references ANR-10-IAHU04-LIRYC; and the European Research Council, H2020 grant agreement number 715093, ECSTATIC.

References

[1] Wang Y, Rudy Y. Application of the method of fundamental solutions to potential-based inverse electrocardiography. Annals of biomedical engineering 2006;34(8):1272–1288.

[2] Karoui A, Bear L, Migerditichan P, Zemzemi N. Evaluation of fifteen algorithms for the resolution of the electrocardiography imaging inverse problem using ex-vivo and in-silico data. Front Physiol 2018;9:1708.

[3] Gulrajani RM, Mailloux GE. A simulation study of the effects of torso inhomogeneities on electrocardiographic potentials, using realistic heart and torso models. Circ Res 1983;52:45–56.

[4] Keller DUJ, Weber FM, Seemann G, Dössel O. Ranking the influence of tissue conductivities on forward-calculated ECGs. IEEE Trans Biomed Eng 2010;57:1568–1576.

[5] Tikhonov AN, Arsenin VI. Solutions of ill-posed problems, volume 14. Winston, Washington, DC, 1977.

[6] Potse M. Scalable and accurate ECG simulation for reaction-diffusion models of the human heart. Front Physiol 2018; 9:370.

[7] Krause D, Potse M, Dickopf T, Krause R, Auricchio A, Prinzen FW. Hybrid parallelization of a large-scale heart model. In Keller R, Kramer D, Weiss JP (eds.), Facing the Multicore-Challenge II, volume 7174 of Lecture Notes in Computer Science. Berlin: Springer, 2012; 120–132.

[8] Barnes JP, Johnston PR. Application of robust generalised cross-validation to the inverse problem of electrocardiology. Computers in biology and medicine 2016;69:213–225.

Address for correspondence:

Pauline Migerditichan
Inria Bordeaux Sud-Ouest,
200 avenue de la vieille tour, 33405 Talence, France.
pauline.migerditichan@inria.fr