A New DDE Smoothing Filter for ECG Signal Denoising

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Abstract

Signal filtering is a challenging problem arising in many applications such as Electrocardiogram (ECG) signal processing. Among the techniques that are used for signal denoising, quadratic variation (QV) regularization and smoothness priors have received significant attention during the past. In this paper, we propose a new approach to smoothing filter design, which is based on a delay differential equation (DDE). The proposed method is generic and can be used for smoothing filter design with different frequency responses. As an example, a specific DDE is used in the regularization term of the optimization algorithm. It depends on the regularization parameter and the delay, where the former is related to the cutoff frequency and the latter is set by user. The DDE smoothing filter was analyzed in the frequency domain. It was shown that smoothness priors and QV regularization are special cases of the DDE smoothing filter when the delay tends to infinity. As an application, the proposed smoothing filter was used for ECG signal denoising over data from the PhysioNet PTB database. The results confirm that the proposed smoothing filter outperforms QV regularization for ECG baseline wander removal.

1. Introduction

The electrocardiogram (ECG) is a bioelectric signal, which records the electrical activity of the heart by means of electrodes attached to the skin. The analysis of ECG proves to be very helpful in explaining the functioning of the heart and identifying various pathological conditions. Hence, it is an important and widely used diagnostic tool. Unfortunately, during ECG recordings, other signals (anything other than the muscular activity of the heart) might pollute the ECG (noise or artifact). Noises are usually originating from external sources such as power line interference, respiration, varying electrode-skin conductance, electronic noise due to nearby instrumentation, and muscle interference [1]. Low frequency interferences (baseline wander or BW) is one of the main artifacts affecting the ECG signal. BW is a low-frequency additive noise and its frequency range is typically less than 0.5 Hz. Unfortunately, BW masks important features of the ECG signal. Hence, the problem of removing ECG baseline wandering for a proper analysis and display is of great importance in biomedical signal processing. That is why the problem of BW removal is the subject of many researches. Among other methods, quadratic variation (QV) regularization or smoothness prioirs have received significant attention. In this paper, we proposed a modification of QV regularization which is based on a delay differential equation (DDE).

2. Method

2.1. Background

The smoothing approach defined by QV regularization or smoothness prioirs [2], is to estimate an unknown signal $x(t)$ from its noisy observation $y(t) = x(t) + v(t)$ using the following least-square estimation (LSE) problem:

$$\underset{x(t)}{\text{argmin}} \int [y(\tau) - x(\tau)]^2 \, d\tau + \lambda \int [D^p x(\tau)]^2 \, d\tau \quad (1)$$

where $D^p x = \frac{d^p}{dt^p} x$ denotes the p-th order derivative of the signal and λ denotes the regularization factor which is used to balance the fidelity term (minimum mean square error) and signal smoothness. In this article, instead of the derivative of the signal, we propose to use a DDE model for the signal as the penalty term in the smoothing problem defined in (1). In the following section the proposed DDE smoothing filter is presented.

2.2. Smoothing filter design using a DDE model

Let us consider the following DDE model

$$\frac{d}{dt} x(t) + \alpha_0 x(t) + \sum_{i=1}^{m} \alpha_i x(t - T_i) = 0, \quad (2)$$

where T_i are time delays and α_i a set of coefficients. By adopting (2) as constraint for $x(\cdot)$ in the regularization term of (1), in place of $D^p x(\tau)$, the desired signal $\hat{x}(t)$ can be estimated by solving

$$\hat{x}(t) = \underset{x(t)}{\operatorname{argmin}} \int [y(\tau) - x(\tau)]^2 \, d\tau + \lambda \int \left[\frac{d}{d\tau} x(\tau) + \alpha_0 x(\tau) + \sum_{i=1}^{m} \alpha_i x(\tau - T_i) \right]^2 d\tau. \tag{3}$$

The proposed optimization problem can be interpreted as a tradeoff between the model accuracy and observation quality. Eq. (3) can be expressed as

$$\hat{x} = \underset{x}{\operatorname{argmin}} \, \| y(t) - x(t) \|^2 + \lambda \, \| f(t) * x(t) \|^2 , \tag{4}$$

where $*$ and $\|\cdot\|^2$ denote the convolution operator and the Euclidean norm, respectively and

$$f(t) = \frac{d}{dt}\delta(t) + \alpha_0 \delta(t) + \sum_{i=1}^{m} \alpha_i \delta(t - T_i),$$

with $\delta(t)$ the Dirac delta function. The optimal solution for (4) is [3]

$$\hat{x}(t) = [\delta(t) + \lambda f(-t) * f(t)]^{\otimes} * y(t), \tag{5}$$

where $[\cdot]^{\otimes}$ denotes the deconvolution operator. The impulse response of the DDE smoothing filter is

$$g(t) = [\delta(t) + \lambda f(-t) * f(t)]^{\otimes} . \tag{6}$$

In the following section, the method is analyzed in the frequency domain.

2.3. Frequency domain analysis

Taking the Laplace transform of Eq. (6), the transfer function of the proposed DDE smoothing filter is

$$G(s) = \frac{1}{1 + \lambda F(s) F(-s)},$$

where

$$F(s) = s + \alpha_0 + \sum_{i=1}^{m} \alpha_i e^{-sT_i}.$$

In the Fourier domain

$$G(\omega) = \frac{1}{1 + \lambda F(\omega) F(-\omega)}$$

where

$$F(\omega) = j\omega + \alpha_0 + \sum_{i=1}^{m} \alpha_i e^{-j\omega T_i}.$$

Note that $G(\omega)$ is a zero-phase (acausal) frequency response.

3. Applications

In this section, a maximally flat and minimum stop-band attenuation smoothing filter is designed.

3.1. Maximally flat and minimum stop-band attenuation smoothing filter

Let us consider the following DDE model, depending on the delay T

$$\frac{d}{dt} x(t) = \frac{1}{T} \left[x\left(t + \frac{T}{2}\right) - x\left(t - \frac{T}{2}\right) \right]. \tag{7}$$

By adopting (7) as constrain for $x(t)$, a new penalty for the solution can be found as

$$\int_a^b \left(\frac{d}{dt} x(\tau) - \frac{1}{T} \left[x\left(\tau + \frac{T}{2}\right) - x\left(\tau - \frac{T}{2}\right) \right] \right)^2 d\tau,$$

while in eq. (5) we have

$$f(t) = \frac{d}{dt}\delta(t) - \frac{1}{T} \left[\delta\left(t + \frac{T}{2}\right) - \delta\left(t - \frac{T}{2}\right) \right],$$

and in the transfer function $F(s) = s - (e^{s\frac{T}{2}} - e^{-s\frac{T}{2}})/T$. Finally, in Fourier domain

$$G_{1,T}(\omega) = \frac{1}{1 + \lambda \left[\omega - \frac{2}{T} \sin\left(\omega \frac{T}{2}\right) \right]^2}. \tag{8}$$

As mentioned before, $G_{1,T}(\omega)$ is a zero-phase frequency response. Suppose that a smoothing filter with a -6 dB cutoff frequency $\omega_c = 2\pi f_c$ is desired. Then the regularization factor, denoted by λ_c, is univocally defined and can be numerically calculated by solving the following equation:

$$\frac{1}{1 + \lambda_c \left(\omega_c - \frac{2}{T} \sin \omega_c \frac{T}{2} \right)^2} = \frac{1}{2},$$

leading to the following optimal value:

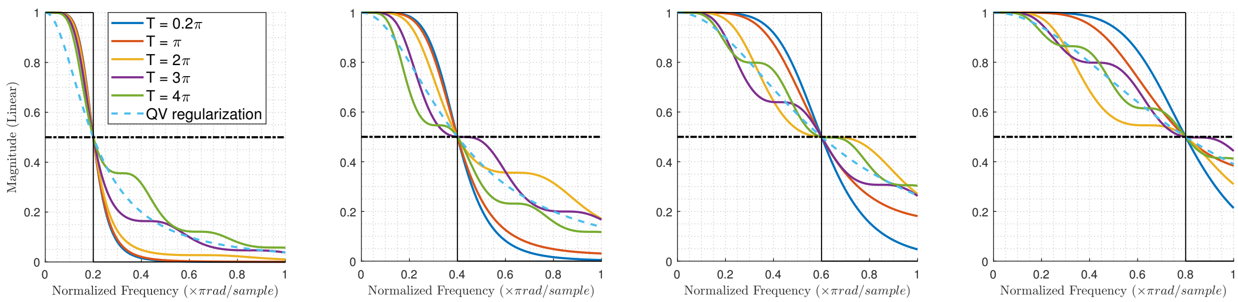
$$\lambda_c = \frac{1}{\left(\omega_c - \frac{2}{T} \sin \omega_c \frac{T}{2} \right)^2}.$$

Finally, the frequency response of the smoothing filter is found by substituting λ_c in (8):

$$G_{1,T}(\omega) = \frac{1}{1 + \left(\dfrac{\omega - \frac{2}{T} \sin \omega \frac{T}{2}}{\omega_c - \frac{2}{T} \sin \omega_c \frac{T}{2}} \right)^2}$$

which depends on the desired cutoff frequency and delay T. It is notable that, when $T \to \infty$, $G(\omega)$ is equal to

$$G_{1,T}(\omega) = \frac{1}{1 + \left(\dfrac{\omega}{\omega_c} \right)^2},$$



(a)$\omega_c = 0.2$. (b)$\omega_c = 0.4$. (c)$\omega_c = 0.6$. (d)$\omega_c = 0.8$.

Figure 1. Amplitude response of the proposed smoothing filter, $G_{1,T}(\omega)$, for different values of T.

which is exactly the frequency response of the first order smoothness priors or QV regularization [4]. The frequency response of the smoothing filter for different values of ω_c is shown in Fig. 1. While the smoothness priors is a special case of the proposed smoothing filter when T tends to infinity, for small values of T, the latter acts better in the pass-band (closer to 1) and stop-band (larger attenuation). Also, the roll-off is faster. These properties can be improved further using p-order differential equations, as in the following DDE model

$$\frac{d^p}{dt^p}x(t) = \frac{1}{T}\frac{d^{p-1}}{dt^{p-1}}\left[x\left(t+\frac{T}{2}\right) - x\left(t-\frac{T}{2}\right)\right]. \quad (9)$$

If we use it as the penalty term in the optimization problem, then the solution (the estimated signal) is

$$\hat{x}(t) = [\delta(t) + \lambda f_p(-t) * f_p(t)]^{\otimes} * y(t),$$

where

$$f_p(t) = D^p\delta(t) - \frac{1}{T}\left[D^{p-1}\delta\left(t+\frac{T}{2}\right) - D^{p-1}\delta\left(t-\frac{T}{2}\right)\right]$$

The transfer function of the smoothing filter is:

$$G_{p,T}(s) = \frac{1}{1+\lambda F_p(s)F_p(-s)},$$

where $F_p(s) = s^{p-1}\left[s - (e^{s\frac{T}{2}} - e^{-s\frac{T}{2}})/T\right]$. In the Fourier domain, the frequency response becomes

$$G_{p,T}(\omega) = \frac{1}{1+\lambda\left(\omega^{p-1}\left[\omega - \frac{2}{T}\sin\omega\frac{T}{2}\right]\right)^2}.$$

The value of λ corresponding to the cutoff frequency ω_c is

$$\lambda_c = \frac{1}{\left(\omega_c^{p-1}\left[\omega_c - \frac{2}{T}\sin\omega_c\frac{T}{2}\right]\right)^2}.$$

leading to

$$G_{p,T}(\omega) = \frac{1}{1+\left[\left(\frac{\omega}{\omega_c}\right)^{p-1}\frac{\omega - \frac{2}{T}\sin\omega\frac{T}{2}}{\omega_c - \frac{2}{T}\sin\omega_c\frac{T}{2}}\right]^2}.$$

As an illustration, the amplitude response of the smoothing filter for $\omega_c = 0.4$ with $p \in (1,\ 2,\ 5,\ 10)$ and different values of T is shown in Fig. 2. In the pass-band, the amplitude response tends to one and in the stop-band it tends to zero as p increases. In other words, the amplitude response tends to the ideal values for large values of p. When $T \to \infty$, the transfer function is equal to

$$G_{p,T}(\omega) = \frac{1}{1+(\omega/\omega_c)^{2p}}.$$

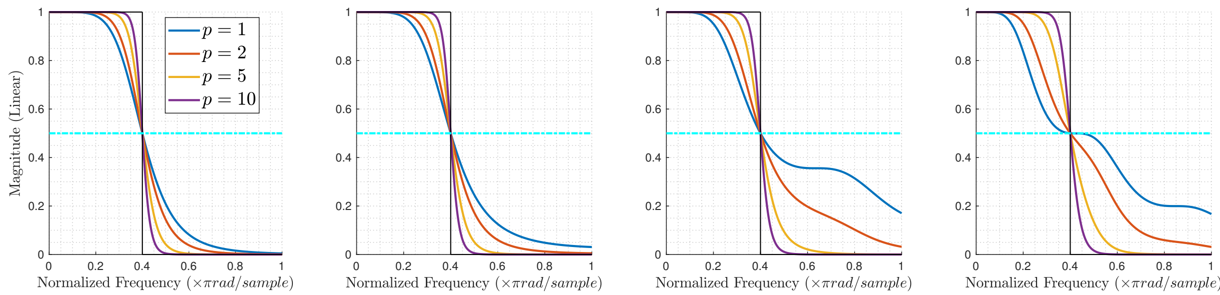
which is exactly the same as the p-th order smoothness priors [4].

3.2. ECG Baseline Wandering Removal

In order to evaluate the performance of the proposed approach, we tested its discrete version for BW removal from ECG signals. The derivations in discrete time can be found in [5]. We employed BW-free ECG records, which were corrupted by a priori known low-frequency noise. In this way, the result of filtering can be compared with the original signal, and the quality of BW removal measured, in terms of the corresponding estimation error, using the NSR defined by:

$$\text{NSR} = \sqrt{\frac{\sum_k (x_k - \hat{x}_k)^2}{\sum_k x_k^2}}, \quad (10)$$

where x_k and $\hat{x}_k$ are the original and estimated signal, respectively. Real ECG signal were obtained from the PhysioNet PTB Diagnostic ECG Database [6], which contains 549 records from 290 subjects. Each record consists of twelve conventional ECG leads plus the three



(a)$T = 0.2\pi$. (b)$T = \pi$. (c)$T = 2\pi$. (d)$T = 3\pi$.

Figure 2. Amplitude response of the proposed smoothing filter, $G_{p,T}(\omega)$, for $\omega_c = 0.4$ and different values of p.

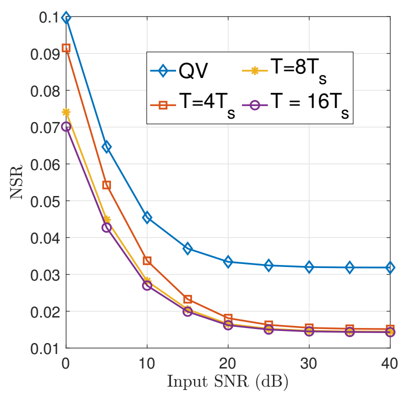


Figure 3. Mean values of NSR for ECG BW removal by QV regularization and the proposed filter, as a function of the power of the baseline noise corrupting the ECG signal.

Frank's ones, sampled at 1kHz with 16-bit resolution. The baseline noises were generated by (11) and then added to each ECG.

$$v(t) = \sum_{i=0}^{10} c_i \cos(2\pi f_i t), \quad (11)$$

where f_i and c_i were randomly selected such that $0 < f_i \leq 0.5$ and $1 \leq c_i \leq 10$. The baseline noise was added with varying SNR (from -10 to 40 dB). We compared our results with those obtained in [2], using QV regularization. The mean of NSR versus different input SNRs, as achieved by our approach and QV regularization are plotted in Fig. 3. The former outperformed the latter.

4. Conclusion

In this paper, a new approach to smoothing filter design was proposed. In the presented approach, a delay differential equation (DDE) was employed as a constraint in a regularized least square optimization framework to estimate the desired signal from its noisy observation. The QV regularization and smoothness priors are special cases of the proposed smoothing filter when the window length tends to infinity. The pass-band flatness, the steepness of the roll-off and the stop-band attenuation of the proposed DDE smoothing filter is controlled by the order of the differential equation (p) and the window length (T). As p increases the amplitude response goes to one in the pass-band and the steepness of the roll-off increases.

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