The Case of Ties in Bubble Entropy

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Bubble entropy (bEn), like Permutation entropy (pEn), belongs in the family of entropy definitions, which exploit sorting for the estimation of the complexity in an *m*-dimensional embedding space. In this paper, we investigate how ties (samples with equal values, very common in HRV series obtained from ECG recorded at sampling rates smaller than 500Hz) affect the method. Although researchers have claimed that in *pEn* ties have no significant effect and might even be beneficial in the context of classification, a common practical work-around is to add small perturbations. Although this breaks ties, it constitute an entropy "leakage" from random noise to the series.

In this work, we quantify how much bEn is affected by ties, analyzing HRV series from two Holter ECG data sets (Normal Sinus Rhythm or NSR, and Congestive Heart Failure or CHF). The original sampling rates are between 128Hz and 250Hz. We further increase the number of ties in the HRV series by duplicating values at random and compare, with statistical ANOVA F-Test, the discriminating capability of the metrics before and after the distortion. We compare bEn, with m=2...50, pEn and sorting entropy (sEn), with m=2...15. Signals were edited with 20%, 30%, 40% and 50% of ties.



F-values for signals with different amount of added ties

An increasing addition of ties inevitably reduces the discriminating capability of bEn, likely due to the loss of information. However, bEn still manages to discriminate the two populations efficiently and almost consistently presents the best F-value among the metric we consider. Surprisingly, pEn and sEnslightly increase their F-value for some values of m, possibly due to an initial low discriminating capability and a subsequent random behavior.