

Ventricular Surface Activation Time Imaging from Clinical Data: A Quantitative Comparison of Two Inverse Approaches

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Abstract

Ventricular activation time maps for both the endocardium and epicardium were estimated from MRI and body surface ECG mapping data which had been acquired under clinical conditions. The associated nonlinear inverse ill-posed problem was solved by two different approaches. The first approach (method A) bases on a linearization technique while the second (method B) employs an optimization routine for nonlinear ill-posed problems. Both methods utilize a second order Tikhonov regularization in order to stabilize the inverse solution. The linearization technique could be demonstrated to perform better, particularly with regard to clinical implementations and applications. In two patients suffering from the Wolff-Parkinson-White syndrome, the activation time maps were estimated and quantitatively compared.

1. Introduction

Endo- and epicardial activation time (AT) imaging from body surface ECG mapping data aims at providing noninvasively the information cardiologists need to localize the origins of cardiac arrhythmias. An estimation of the AT map can be obtained by solving a nonlinear inverse ill-posed problem [1]. Tackling this kind of problems requires, in general, several consecutive steps to be carried out. One important step involves the regularization of the inverse ill-posed problem [2, 3]. Mathematically, regularization causes the inverse of the compact lead field operator to become continuous. Technically, regularization can be achieved by appending a so-called smoothing term to the cost function formulation. In many cases, the Laplacian operator acts as smoothing operator [4]. The amount of regularization is thereby controlled by a regularization parameter. In other words, the regularization parameter controls the weight of the Laplacian operator with regard to the lead field operator. The proper value for this parameter can be gained by employing the L-curve

method [5]. For nonlinear inverse problems, however, the regularization procedure and the determination of the regularization parameter show insufficient performance in terms of feasibility for clinical implementations and applications. This is mainly caused by the fact that the L-curve is not shaped like a typical L-curve. For this reason, it is necessary to have an easier-to-handle regularization procedure at hand.

In this work we demonstrate that a linearized formulation of the underlying nonlinear inverse ill-posed problem shows a better performance with respect to the proper amount of regularization and is therefore more suitable for technical implementations. To this end, ventricular surface AT maps are estimated and quantitatively compared in two patients suffering from the WPW syndrome. The AT map is thereby estimated from body surface ECG recordings related to the WPW syndrome and a right ventricular pacing protocol.

2. ECG, MRI, and modeling

The body surface ECG was recorded with the Mark-8 body surface potential mapping system (Biosemi V.O.F., The Netherlands). The Mark-8 system is an on-line portable computer acquisition system with data transmission by optical fiber. A radio-transparent carbon electrode array (University of Amsterdam, The Netherlands) was utilized to record unipolar ECG data from 62 torso sites. Sampling rate was 2048Hz per channel, A/D resolution was 16 bit, and an analog high pass and low pass filter of 0.32 and 400Hz was used. The 62 electrode and 7 reference positions were acquired with the FASTRAK system (Polhemus Inc., USA).

The torso was imaged with a 1.5Tesla MR scanner (Siemens Vision Plus). The myocardium was additionally imaged in an ECG-gated breath-hold oblique imaging mode in order to model the heart's surface. Vitamin E markers were utilized to determine 7 reference positions from the axial MR scans to be able to couple MRI with ECG data. The entire volume conductor including the blood mass was

modeled with 4098 triangles [6]. The ventricular surface is represented by 1026 elements and 515 nodal points. The different compartments comprise the torso, the lungs, and the blood mass. The associated conductivities are 0.2, 0.08, and 0.6Sm^{-1} , respectively.

3. AT map and the inverse problem

The AT map on the ventricular endo- and epicardium is estimated by employing two methods capable of solving nonlinear inverse ill-posed problems. These two methods are denoted by method A and B, and are described below. Both methods depend strongly on high-quality starting AT maps to yield satisfying results. This starting AT map is obtained by applying the critical point theorem [7].

3.1. Method A

The relation between activation time on the ventricular surface and the body surface ECG is represented by a nonlinear inverse ill-posed problem. It can be formulated in the following form:

$$\mathcal{F}\tau = D, \quad (1)$$

where \mathcal{F} is a nonlinear operator which maps the activation time τ onto the body surface ECG data D . Assuming $\{\tau_k\}_{k=0}^{\infty}$ to be a series of approximations of the true solution τ and linearizing Eq. (1) in the point τ_k yields

$$\mathcal{F}\tau_k + \mathcal{F}_k(\tau - \tau_k) = D, \quad (2)$$

where \mathcal{F}_k is an abbreviation for $\mathcal{F}(\tau_k)$ and \mathcal{F} represents the Frechet derivative of the operator \mathcal{F} . Equation (1) can be written in a technically more useful form

$$\mathcal{F}_k\tau = D_k, \quad (3)$$

with $D_k = D + \mathcal{F}_k\tau_k - \mathcal{F}\tau_k$. Equation (3) is, in general, again ill-posed. In order to find a regularized approximation for τ a regularization method for linear ill-posed problems can be employed. Applying second order Tikhonov regularization with the Laplacian operator Δ and the regularization parameter λ_k yields the following regularized approximation

$$\tau = (\mathcal{F}_k^*\mathcal{F}_k + \lambda_k^2\Delta^*\Delta)^{-1}\mathcal{F}_k^*D_k, \quad (4)$$

where the asterisk marks the adjoint operator. By repeating this process an iteration method is obtained

$$\tau_{k+1} = \tau_k + \delta\tau_k \quad (5)$$

with the incremental activation time

$$\delta\tau_k = (\mathcal{F}_k^*\mathcal{F}_k + \lambda_k^2\Delta^*\Delta)^{-1} \times \{\mathcal{F}_k^*(D - \mathcal{F}\tau_k) - \lambda_k^2\Delta^*\Delta\tau_k\}. \quad (6)$$

As the index k increases the iteration process in Eq. (5) converges to a regularized approximation of the AT pattern τ . This iteration process is equivalent to the Gauss–Newton method for minimizing the cost function Ψ in method B. The limit of the iteration represents a solution of the corresponding Euler equation of this cost function [3]. It is thus possible to extend regularization methods from linear to nonlinear inverse ill-posed problems.

3.2. Method B

This method simply implements an optimization routine (E04UCF, NAG Ltd., UK) for nonlinear problems. This routine is then employed to minimize the cost function [3, 8]

$$\Psi = \int_0^T \|\mathcal{L}\phi_m(t, \tau) - D(t)\|_2^2 dt + \lambda^2 \|\Delta\tau\|_2^2, \quad (7)$$

which is composed of an integral over the residual norm plus a regularization term. The transfer or lead field matrix \mathcal{L} describes the relation between the transmembrane potential ϕ_m on the endo- and epicardium and the body surface ECG data D . The time interval $[0, T]$ corresponds to the duration of the ventricular depolarization sequence. As in method A, a second order Tikhonov regularization approach with the Laplacian operator is employed to stabilize the inverse solution of Eq. (7) in terms of measurement noise and errors in modeling the patients geometry. The regularization parameter λ specifies the weight between the residual norm $\|\mathcal{L}\phi_m(t, \tau) - D(t)\|_2$ and the regularizing smoothing norm $\|\Delta\tau\|_2$. The determination of λ is again based on the L-curve method. The transmembrane potential ϕ_m is described at each source point of the ventricular surface by the analytical formula

$$\phi_m(t, \tau) = \frac{u}{2} \left\{ 1 + \frac{2}{\pi} \arctan \left[\pi \frac{t - \tau}{w} \right] \right\} + a, \quad (8)$$

with the resting membrane potential $a = -90\text{mV}$, the action potential amplitude $u = 100\text{mV}$, and the rise time $w = 2\text{ms}$.

4. Results

The AT map in two patients was estimated by applying method A and B. Both patients had been suffering from the WPW syndrome and were scheduled for catheter ablation. A pacing protocol was performed before catheter ablation. The pacing site was, according to the cardiologist, close to the apex of the right ventricle. The associated reconstructed AT map is depicted in Fig. 1. Both methods localized the

Table 1. Correlation coefficients (CC) and relative root mean square errors (relRMS) between the estimated AT maps in both patients for method A and method B.

	Pacing Protocol		WPW syndrome	
	CC	relRMS	CC	relRMS
Patient 1	0.97	8.7%	0.92	8.0%
Patient 2	0.92	19%	0.77	19%

pacing site fairly accurately. Table 1 provides a quantitative comparison of the estimated AT maps. The reconstructed AT map displaying the WPW syndrome can be seen in Fig. 4. Both methods were able to localize the accessory pathway inside the region of the right ventricular out-flow tract. The associated L-curves in determining the AT map are depicted in Figs. 2 and 3. The advantage of method A can be seen clearly. At each iteration step the L-curve is really shaped like an L-curve. The proper regularization parameter can be found easily at each step. In method B it is more difficult to find the proper value for the regularization parameter because the L-curve does not display its typical shape. A close look at Table 1 reveals that the results in patients one are much better than in patient 2. In particular, movements of patient 2 during the MRI scan caused the coupling of MRI and ECG mapping data to be of minor quality.

5. Conclusion

This work investigated and compared the technical feasibility of method A and B. Both methods are capable of estimating AT maps from body surface ECG mapping data. From a mathematical point of view both methods are equivalent. On a purely technical ground, however, the regularization procedure of method A can be performed more clearly and straightforwardly in terms of measurement noise, errors in modeling, and errors in coupling MRI and body surface ECG mapping data.

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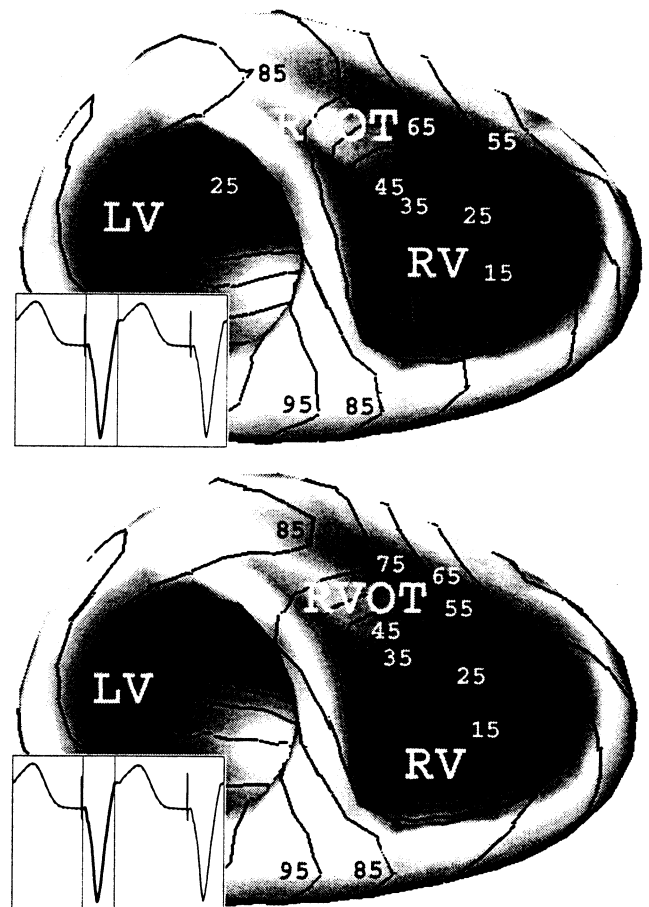


Figure 1. AT map according to a pacing protocol in the right ventricle in patient 1. AT estimated via method A (upper panel) and method B (lower panel). RV, LV, and RVOT indicates right ventricle, left ventricle, and right ventricular out-flow tract, respectively.

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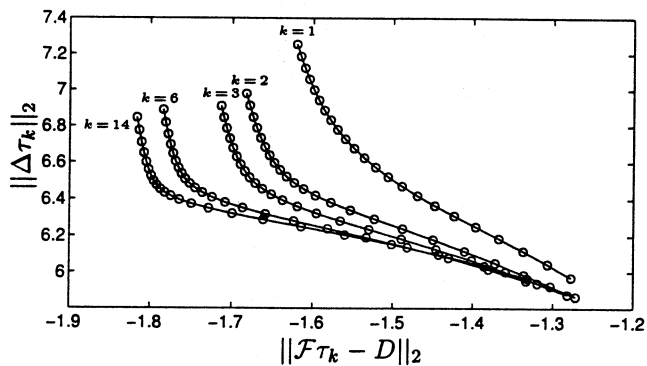


Figure 2. L-curves plot of method A.

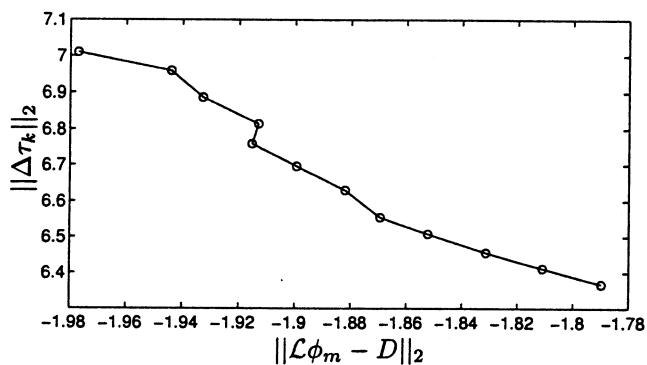


Figure 3. L-curve plot of method B.

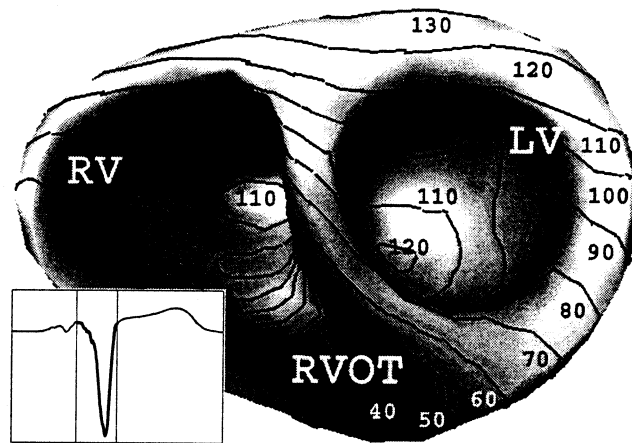
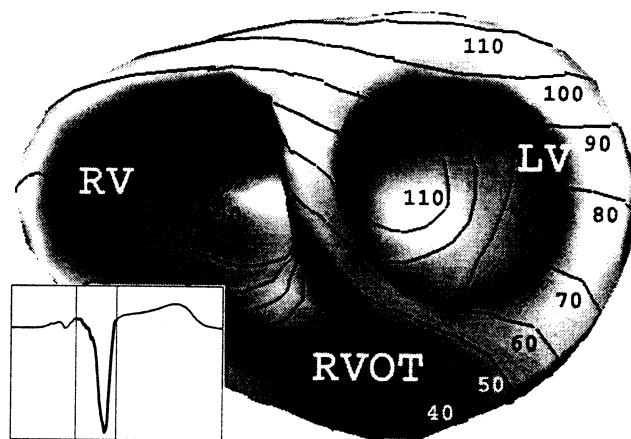


Figure 4. AT map of patient 1 suffering from the WPW syndrome estimated via method A (upper panel) and method B (lower panel). RV, LV, and RVOT indicates right ventricle, left ventricle, and right ventricular out-flow tract, respectively.

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