On the Numerical Analysis of Coronary Artery Wall Shear Stress

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Abstract

A numerical analysis is performed in order to study coronary artery wall shear stress and its dependence on i) unsteadiness of the flow, ii) motion and deformation of the vessel geometry and iii) shear thinning properties of blood. An instationary geometric model of the right coronary artery is developed from intravascular ultrasound measurements complemented with the 3-D reconstructed measurement location and orientation. The velocity distribution in the model is computed by solving the ALE formulation of the Navier Stokes equations using a finite element method. The averaged WSS pattern corresponding to time dependent shear thinning flow in the instationary model is compared to the pattern corresponding to steady mean Newtonian flow in the rigid end-diastolic model.

1. Introduction

Intimal thickening is an integral part of the development of atherosclerosis. It has been shown that the location of intimal thickening correlates to low and oscillating wall shear stress (WSS) [1]. The WSS is influenced by local hemodynamics which are dominated by geometric effects [2]. For the atherosclerotic right coronary artery (RCA), Krams et al. have demonstrated the inverse relation between the intimal thickness and WSS in a numerical patient study [3]. The model that was applied comprised steady Newtonian flow in a rigid, anatomically realistic model of the end-diastolic (ED) RCA. In vivo however, the flow in the RCA and the vessel geometry are time dependent by nature due to the contraction cycle of the heart and the resulting periodic pressure wave. Furthermore, the rheological behavior of blood is shear-rate dependent [4]. In order to study the validity of the assumptions made by Krams et al., the current study presents a numerical analysis of the WSS of unsteady shear thinning flow in a moving and deforming model of the RCA. The time-averaged WSS pattern in the instationary model is compared to the pattern corresponding to steady mean Newtonian flow in the rigid ED model. Also the effect of the different viscosity models that can be applied is evaluated.

2. Methods

An instationary model of the RCA geometry is developed from end-diastolic (ED) and end-systolic (ES) intravascular ultrasound measurements (IVUS) complemented with the 3-D reconstructed measurement location and orientation of the IVUS catheter [5] (see Figure 1).

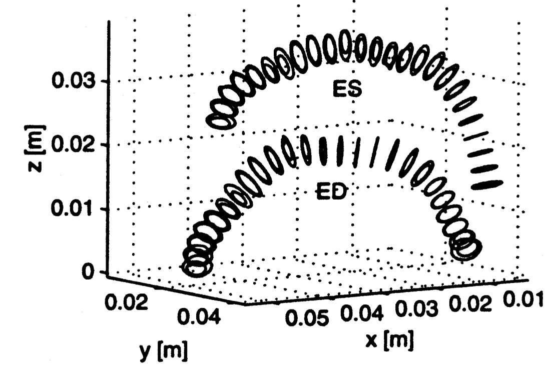

Figure 1. Measured geometry of the RCA (lumen and wall) at the end-diastolic (ED) and end-systolic (ES) position; distal end to the left.

The geometry as a function of time is determined by interpolation between the two depicted states using the cubic root of the ventricular volume curve as the interpolation function. At the proximal flow boundary, a time dependent flat velocity profile is prescribed corresponding to a physiologically realistic flow rate (see Figure 2).

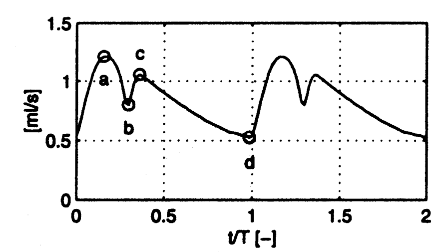

Figure 2. Right coronary flow rate, after [6].

The velocity distribution in the model is computed

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by solving the Arbitrary Lagrangian Eulerian (ALE) formulation of the Navier-Stokes equations:

$$\begin{cases} M\dot{v} + N(v-w)v + S(v)v - L^T p &= f \\ Lv &= 0 \end{cases} \quad (1)$$

with v and w the respective fluid and mesh velocity vectors, M the mass matrix, N the convective part and S the viscous part of the stiffness matrix, L the gradient matrix and f the external stress vector. All vectors and matrices are time dependent. In the non-Newtonian case, the shear thinning behavior of blood is modeled using a Carreau-Yasuda model:

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\lambda\dot{\gamma})^a]^{\frac{n-1}{a}} \quad (2)$$

with $\dot{\gamma}$ the shear rate and η_0, η_∞, n, λ and a rheological parameters for the viscosity. The shear thinning behavior of blood can be approximated by a Newtonian model by scaling the viscosity to a characteristic shear rate. In this case, the above equation is applied to the average shear rate in a 2-D Poiseuille flow: $\dot{\gamma}_c = 4U/D$, with U the average velocity and D the mean diameter [7]. The resulting viscosity value is higher than in the unscaled situation. The shear thinning viscosity curve and the scaled Newtonian approximation for the RCA model are shown in Figure 3.

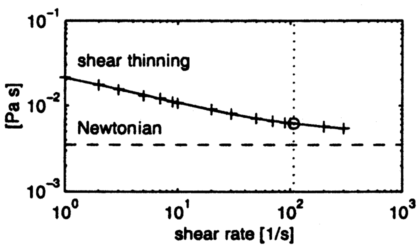

Figure 3. Shear rate dependent viscosity of blood and scaled Newtonian approximation for the RCA model (solid: shear thinning; dashed: unscaled Newtonian; circle: scaled Newtonian), after [4].

3. Results

In the initial part of this study, the effects of the viscosity models and vessel motion and deformation on the velocity patterns are evaluated using a simplified geometry of the RCA. First, the impact of the viscosity models is studied using steady flow in a rigid curved tube approximation of the ED state. The flow rate is based on the time-averaged mean of the flow rate depicted in Figure 2. The results are shown in Figure 4.

Between unscaled, scaled and shear thinning solutions there are small differences in the axial and secondary flow

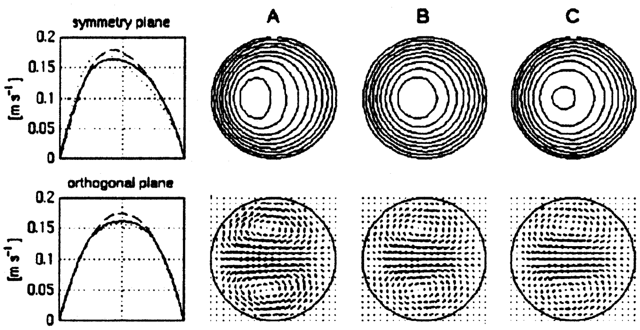

Figure 4. Axial and secondary velocity distributions of steady flow in a rigid curved tube approximation of the RCA (dotted: A-unscaled Newtonian; dashed: B-scaled Newtonian; solid: C-shear thinning).

distributions. The viscosity-scaled situation provides a better approximation of the shear thinning case than the unscaled solution.

Secondly, the effect of vessel motion and deformation is assessed by comparing the results of unsteady shear thinning flows through rigid ED and time dependent curved tube approximations of the RCA geometry. The corresponding velocity patterns are shown in Figure 5. The four instants of time that are depicted are characterized by local maximum and minimum values of the flow rate, as indicated in Figure 2.

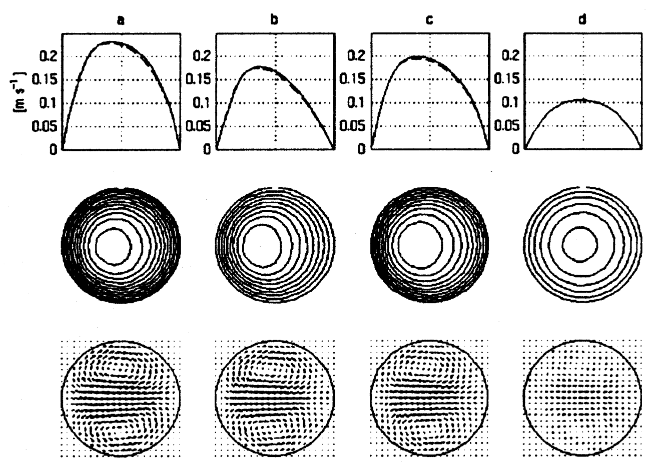

Figure 5. Axial and secondary velocity distributions of unsteady flow in a moving and deforming curved tube approximation of the RCA (dashed: rigid).

The differences between rigid and moving vessel cases are small. In the first three instants, the velocities in the moving geometry are slightly larger than in the rigid case. In the fourth time instant, when the dynamic model is in the diastolic state, the velocity pattern in the moving and deforming geometry is correctly predicted by the rigid case. Because of this, the small variations seem to be driven by

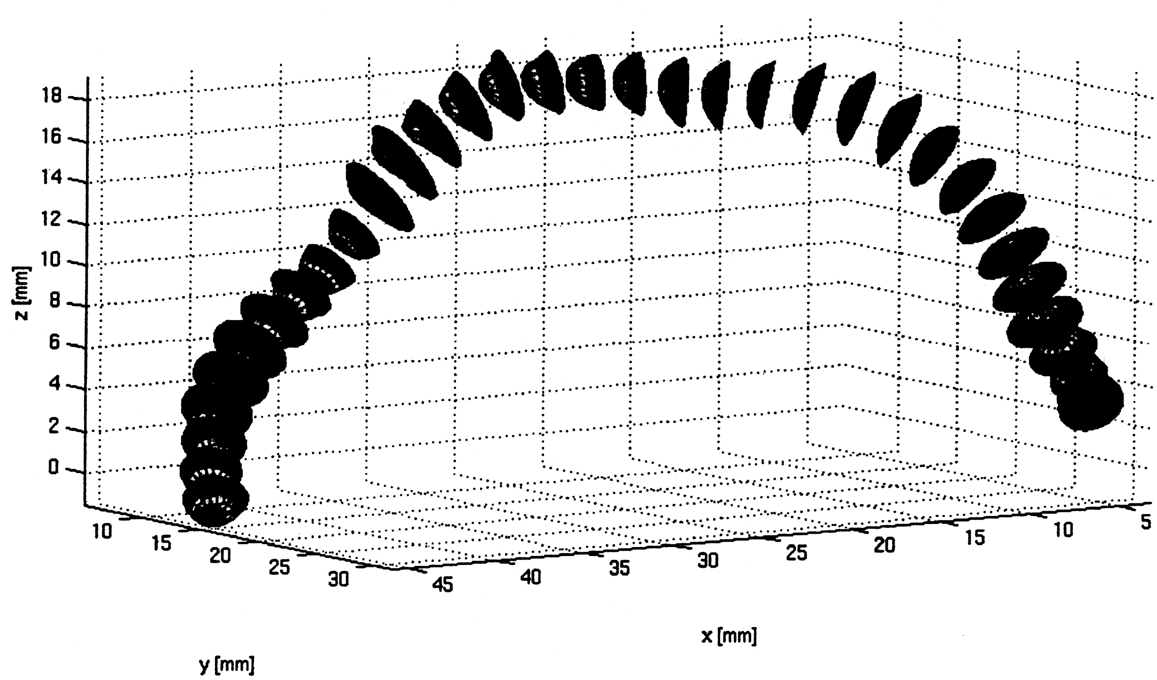

Figure 6. Velocity distribution of steady scaled Newtonian flow in the rigid ED geometry of the RCA.

the geometrical changes that occur between the diastolic and systolic states rather than by the time dependence of the model.

In the second part of this study, the WSS distribution in the real RCA geometry is computed. The effects of unsteadiness of the flow and vessel motion and deformation are evaluated by comparing the time-averaged WSS of shear thinning flow in the dynamic model to the WSS of steady scaled Newtonian flow in the rigid ED geometry. The steady velocity distribution in the rigid geometry is shown in Figure 6. The WSS patterns corresponding to both the stationary and dynamic cases are shown in Figure 7.

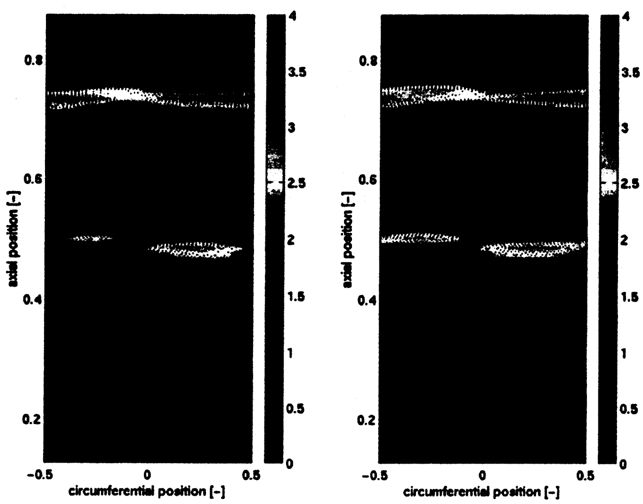

Figure 7. WSS distribution in the RCA: steady mean (left) vs. unsteady time-average (right).

From Figures 6 and 7 it can be gathered that both the velocity distribution and the WSS pattern in the RCA are highly dependent on local geometric properties of the artery. This is most clearly indicated by the high velocities and WSS values that occur at around 75 percent of the artery length. Due to a stenosis and the resulting sudden decrease of the lumen, locally the fluid is accelerated and the WSS is increased. These variations however are present in both the stationary and dynamic patterns and will therefore not influence the correlation with artery wall thickness as demonstrated by [3]. From Figure 7 it is concluded that the time-averaged WSS pattern in the dynamic case is well approximated by the steady mean WSS pattern in the rigid geometry.

4. Conclusions

A comparison of the velocity distributions corresponding to Newtonian and shear thinning flows in the RCA shows that scaling of the viscosity in the Newtonian case is required to approximate the shear thinning behavior of blood. A comparison of unsteady shear thinning flows in the rigid ED and dynamic models shows that the velocity distribution in the RCA is not greatly affected by motion and deformation of the artery. The time-averaged WSS distribution in the RCA can be well approximated by modelling steady mean Newtonian flow in the rigid diastolic geometry using a scaled viscosity.

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