Estimation of Global Parameters for the Analysis of Left Ventricular Motion

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Abstract

A method for the extraction of global parameters such as rotation and deformation matrices and translation vector, for left ventricular (LV) motion analysis in a three-dimensional (3D) space, is presented. The global parameters are estimated from the analysis of 3D echocardiographic sequences.

First step is the segmentation of LV chamber applying level set technique. Second step consists in applying the Umeyama theorem to two LV frames of a cardiac cine-loop: the outputs of this step are the translation vector and the rotation and deformation matrices that characterize the motion between the two analyzed frames. From these parameters the principal axes of deformation, the deformation modules, the rotation angles and also the percent volume variation between the two frames can be calculated. The results obtained on phantoms demonstrate that the algorithm for the motion analysis is correct. This preliminary study also shows that on 3D echocardiographic data the results are acceptable.

1. Introduction

Echography is one of the most widely used diagnostic techniques for cardiac imaging. The traditional transthoracic modality allows to perform an analysis of the morphology of the cardiac structures such as cavities, cardiac valves and big vessels. Besides cardiac motion estimation is very important in understanding cardiac dynamics and in noninvasive diagnosis of heart disease. The quantification of global parameters to evaluate LV motion and contractility, wall thickness, diskinetic and akinetic area extension, degree of ischemia and infarction and aneurysm presence would be useful for the management of patients affected by "dynamic" diseases as ischemic cardiomyopathy. Actually an accepted method for the analysis of LV motion field does not exist. In literature two classes of algorithms to study ventricular wall motion are proposed. The first class is based on optical flow techniques. The quantification of heart motion is based on the computation of the 3D vector field associated with the non-rigid motion of the time-varying brightness of a sequence of cardiac echo data. Different approaches are possible to calculate the optical flow [1-3]. The second class is based on geometric algorithms that is, starting from a manual or automatic edge detection, for each image of the sequence, the motion field is identified applying geometric methods as the equidivision method [4], the radial method [5], the centerline method [6] and in a 3D domain the centersurface method [7].

The cited methods are affected by many problems concerning low data resolution, noise presence, observer relative orientation respect to the heart (orientation that changes during the cardiac cine-loop), hypothesis on the geometry of the ventricle, not valid in pathological conditions. Besides, the analysis of the movement of left ventricle is influenced by the fact that the heart is free to rotate and translate inside the chest, also following movements due to respiration. To compensate the movement of the heart not directly due to contractility, different types of data realignment have been proposed but the problem is still open.

In this study we propose the application of the least-squares estimation of transformation parameters between two point patterns [8] represented by two endocardial surfaces. The two endocardial surfaces are obtained applying a segmentation technique based on level set models [9-13] to real-time 3D echocardiographic data (RT3DE) [14,15].

2. Method

The method for the extraction of global parameters for LV motion field analysis is divided in two steps described in the following paragraphs.

2.1. Segmentation procedure

The LV endocardial surface has been segmented applying the level set formulation of interface motion [9] as described in [12]. The result of this segmentation step is a 3D array in which the position of the zero-crossing values represents the endocardial surface location. The distance between the two estimated surfaces has been calculated: for each point in the second frame the point at minimum distance in the first frame is looked for. The coordinates of these two points represent the spatial coordinates respect to the ultrasound transducer position and refer to endocardial surface location and they have been stored in two matrices.

2.2. Estimation of global parameters

Let's be J and K the two $3\times n$ matrices representing two patterns of n points. The problem is to find the similarity transformation parameters between these two patterns giving the minimum value of the mean square error. This problem is called absolute orientation problem and it can be solved in a 3D domain using both iterative and non-iterative algorithms. A strict solution, based on the singular value decomposition of a covariance matrix of the data presented in [16], is given by Umeyama [8].

The mean square error e^2 is defined as

$$e^2(R,\mathbf{t})=\frac{1}{n}\sum_{i=1}^{n}\left\|\mathbf{k}_i-(R\mathbf{j}_i+\mathbf{t})\right\|^2$$

and the minimum value ε^2 of e^2 of the two patterns with respect to the similarity transformation parameters (R is the rotation matrix, $\mathbf{t}$ is the translation vector, D is a diagonal matrix and $\mathbf{j}_i$ and $\mathbf{k}_i$ with i=1..n are the sets of points of J and K) is:

$$\varepsilon^2=\sigma_k^2-\frac{tr(DS)^2}{\sigma_j^2}$$

where:

$$\boldsymbol{\mu}_j=\frac{1}{n}\sum_{i=1}^{n}\mathbf{j}_i,\quad \boldsymbol{\mu}_k=\frac{1}{n}\sum_{i=1}^{n}\mathbf{k}_i,$$

$$\sigma_j^2=\frac{1}{n}\sum_{i=1}^{n}\left\|\mathbf{j}_i-\boldsymbol{\mu}_j\right\|^2,\quad \sigma_k^2=\frac{1}{n}\sum_{i=1}^{n}\left\|\mathbf{k}_i-\boldsymbol{\mu}_k\right\|^2,$$

$$\sum\nolimits_{jk}=\frac{1}{n}\sum_{i=1}^{n}(\mathbf{k}_i-\boldsymbol{\mu}_k)(\mathbf{j}_i-\boldsymbol{\mu}_j)^T$$

and let a singular value decomposition of $\sum_{jk}$ be UDV^T ($D=diag(d_i)$, $d_1\geq d_2\geq d_3\geq 0$) and

$$S=\begin{cases} I & if\ \det(\sum_{jk})\geq 0 \\ diag(1,1,-1) & if\ \det(\sum_{jk})<0 \end{cases}$$

$\sum_{jk}$ is a covariance matrix of J and K, $\boldsymbol{\mu}_j$ and $\boldsymbol{\mu}_k$ are the mean vectors of J and K, and σ_j^2 and σ_k^2 are the variance around the mean vectors of J and K respectively.

If the *rank* ($\sum_{jk}$)$\geq$2 the optimum transformation parameters are:

$$R=USV^T$$
$$\mathbf{t}=\boldsymbol{\mu}_k-R\boldsymbol{\mu}_j$$

For a complete proof of the statement see Umeyama [8].

This theorem always gives the correct transformation parameters even when the data are corrupted.

The two parameters give information about the roto-translation motion. If the values in the matrix R are known, the rotation angles α, β, and γ respect to x, y and z axes respectively can be obtained by the following representation of R:

$$\begin{bmatrix} \cos\gamma\cos\beta & \sin\gamma\cos\beta & -\sin\beta \\ -\sin\gamma\cos\alpha+\cos\gamma\sin\beta\sin\alpha & \cos\gamma\cos\alpha+\sin\gamma\sin\beta\sin\alpha & \cos\beta\sin\alpha \\ \sin\gamma\sin\alpha+\cos\gamma\sin\beta\cos\alpha & -\cos\gamma\sin\alpha+\sin\gamma\sin\beta\cos\alpha & \cos\beta\cos\alpha \end{bmatrix}$$

The 3×3 matrix that deforms the pattern represented by the matrix J in the matrix K after the roto-translation motion can be expressed as a second order tensor in which the simmetric part is the deformation matrix D we are looking for [17]. The eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of the matrix D represent the principal deformation axes and the corresponding eigenvalues λ_1, λ_2, λ_3 are the deformation ratios along the principal axes.

The volume variation between J and K can be expressed by:

$$\frac{\Delta V}{V}=\frac{V_K-V_J}{V_J}=\lambda_1\lambda_2\lambda_3-1$$

Therefore from this analysis, to describe motion in a 3D domain, we obtain the following global parameters:

- R rotation matrix;
- α, β, γ rotation angles respect to the x, y and z axes;
- $\mathbf{t}$ translation vector;
- D deformation matrix;
- $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ principal deformation axes;
- λ_1, λ_2, λ_3 deformation ratio along the principal axes;
- $\frac{\Delta V}{V}$ percent volume variation.

In our study the matrices J and K are represented by the endocardial surfaces described by n points and referring to two different frames of the cardiac cine-loop.

3. Results

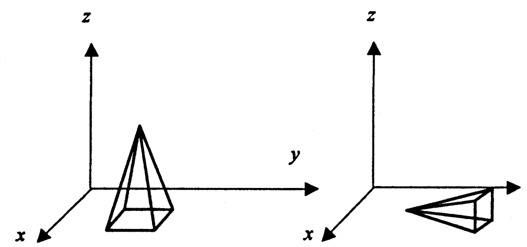
In this section the results obtained applying the model on synthetic and RT3DE data are presented.

To test the model a known roto-translation and deformation motion has been applied to different kind of synthetic 3D objects (cubes, spheres and pyramids). For all the cases the difference between the final configuration K and the configuration obtained applying the transformation $D\times[R\times J+\mathbf{t}]$ is zero. Besides the computed errors on deformation ratio along the reference

Table 1. Comparison between the results obtained applying an a priori known motion of 3D object.

Motion	n	Obtained results				Expected results			
		λ_1	λ_2	λ_3	ΔV/V	λ_1	λ_2	λ_3	ΔV/V
Cube rototranslation	8	1	1	1	0	1	1	1	0
Cube deformation	8	7	7	7	342	7	7	7	342
Cube rototranslation and deformation	8	0.512	0.863	1	-0.558	0.5	1	1	-0.5
Pyramid rototranslation	5	1	1	1	0	1	1	1	0
Pyramid rototranslation and deformation	5	0.8	0.8	0.8	-0.488	0.8	0.8	0.8	-0.488
Sphere contraction	31224	0.917	0.917	0.917	-0.2298	0.917	0.917	0.917	-0.23

axes and on rotation angles were zero (Table 1). An example is presented in Figure 1.



$$J=\begin{bmatrix}2 & 1 & 1 & 2 & 1.5\\ 2 & 2 & 1 & 1 & 1.5\\ 0 & 0 & 0 & 0 & 6\end{bmatrix} \quad K=\begin{bmatrix}2.9 & 2.1 & 2.1 & 2.9 & 2.5\\ 7 & 7 & 7 & 7 & 2.2\\ 0.9 & 0.9 & 0.1 & 0.1 & 0.5\end{bmatrix}$$

$$R=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix} \quad \mathbf{t}=\begin{bmatrix}1.3\\ 7\\ -0.7\end{bmatrix} \quad D=\begin{bmatrix}0.8 & 0 & 0\\ 0 & 0.8 & 0\\ 0 & 0 & 0.8\end{bmatrix}$$

$$\lambda_1=0.8 \quad \lambda_2=0.8 \quad \lambda_3=0.8 \quad \frac{\Delta V}{V}=-0.488$$

$$\mathbf{v}_1=[-0.0497 \quad -0.7011 \quad 0.7113]$$
$$\mathbf{v}_2=[0.5629 \quad 0.5687 \quad -0.4303]$$
$$\mathbf{v}_3=[0.8250 \quad -0.4303 \quad -0.3663]$$
$$\alpha=90°,\ \beta=0°,\ \gamma=0°$$

Figure 1. The roto-translation and deformation of a pyramid: the estimated parameters.

The regression line between the real and estimated volumes variation is $y=x$ -0.01, r=1.

The model has been also tested on RT3DE data belonging to a cardiac cine-loop. The percent volume variation values between two frames obtained by our model have been compared with those obtained by segmentation technique based on level set models (Table 2).

Table 2. Frame by frame comparison of percent volume variation during systolic phase; ε^2 is the least mean square error.

	n	ΔV/V% obtained	ΔV/V% expected	ε^2
Frames 1-2	18718	-8.97	-6.4	1.5
Frames 2-3	17516	-10.93	-10.15	1.3
Frames 3-4	16932	-25.4	-22.2	1.8
Frames 4-5	13634	-3.17	-5.22	1.4
Frames 5-6	13812	-6.9	-7.65	1.9
Frames 6-7	13038	-2.88	-1.62	1
Frames 7-8	12984	-29.6	-33.11	1

The regression line was $y = 0.93x+1.09$ and the correlation coefficient was r = 0.98 (Figure 2).

For a small set of RT3DE data the ejection fraction (EF) values obtained with this model and with magnetic resonance imaging (MRI) have been compared (Figure 2).

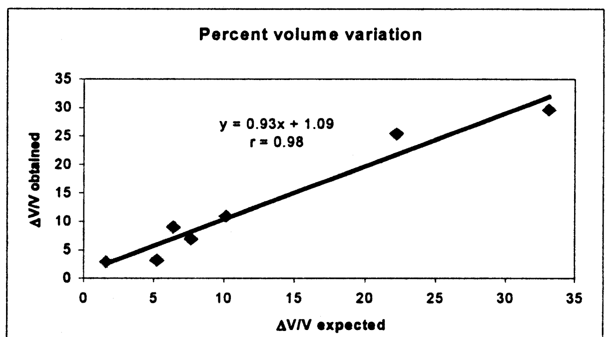


Figure 2. Correlation between obtained and expected percent volume variation values.

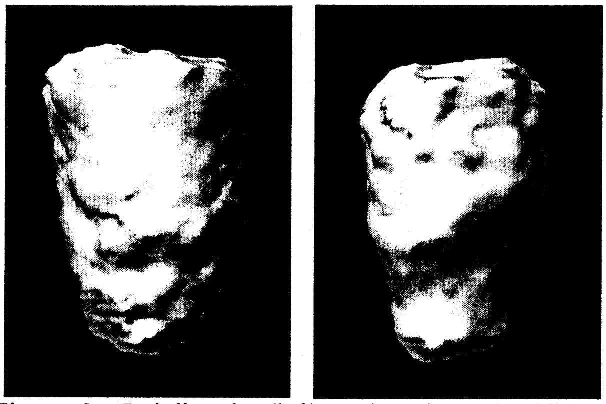


Figure 3. End-diastole (left) and end-systole (right) endocardial surfaces of RT3DE data. EF values obtained by our segmentation method is EF_{RT3DE} =18.3% and by MRI data is EF_{MRI}=24.3%.

4. Conclusion

A method based on least square estimation of transformation parameters between two point patterns has been applied to evaluate LV motion analysis.

The results obtained on phantoms demonstrate that the algorithm for the motion analysis is correct (r=1). This preliminary study also shows that on real-time 3D echocardiographic data the results are acceptable. The errors obtained on real data can be explained considering two different reasons: the non perfect point to point correspondence between the two surfaces and the error that affects the LV volume values obtained applying MRI or level set based segmentation. Actually the correspondence between the two surfaces is calculated considering a rotational free motion field.

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