Multifractal Analysis of Heart Rate Variability

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Abstract

We have studied the RR-variability of 5 young and 5 elderly subjects for multifractal behaviour using a partition function method. The partition function, obtained using the wavelet transform modulus maxima method, allowed the computation of local Hurst exponents. The multifractal spectrum was obtained from the Hurst exponents using a Legendre transformation. The computations revealed specific values of the Hurst exponent at which the multifractal spectrum peaked, with a clear separation between young and elderly subjects. The peaks of the young subjects occurred for Hurst exponents in the range 0.06 to 0.13 whereas the peaks for the elderly subjects were in the range 0.29 to 0.36.

1. Introduction

It is widely accepted that scaling [1] concepts can lead to a profound description and categorisation of a wide range of highly complex physical and biological phenomena, including heart rate variability. Moreover, it is possible that the intricacies of heart rate variability can be unraveled further using multifractal [2, 3] scaling analysis. The basis for associating heart rate variability with multifractal scaling is the self-similar structure of the variability. The expectations are that as the heart becomes less healthy, for example through aging, it gradually loses its adaptability of response and shows less complexity, and that the quantification of this change using multifractal scaling analysis could improve the portfolio of non-invasive tools of analysis for use in clinical cardiology.

In this study, the *RR* variability in ECG data from 5 young and 5 elderly subjects was analysed for multifractal behaviour using a partition function [4] method. The aims were to study how the variability is characterized using a spectrum of scaling indices and to quantify the change that occurs in the variability with age.

2. Methods

2.1. Subjects

About 6000 *RR* intervals were used from each of 5 young and 5 elderly subjects. The data for this initial study was available from the PhysioNet Database [5]. The *RR* intervals from a typical young subject and a typical elderly subject are shown in Fig. 1.

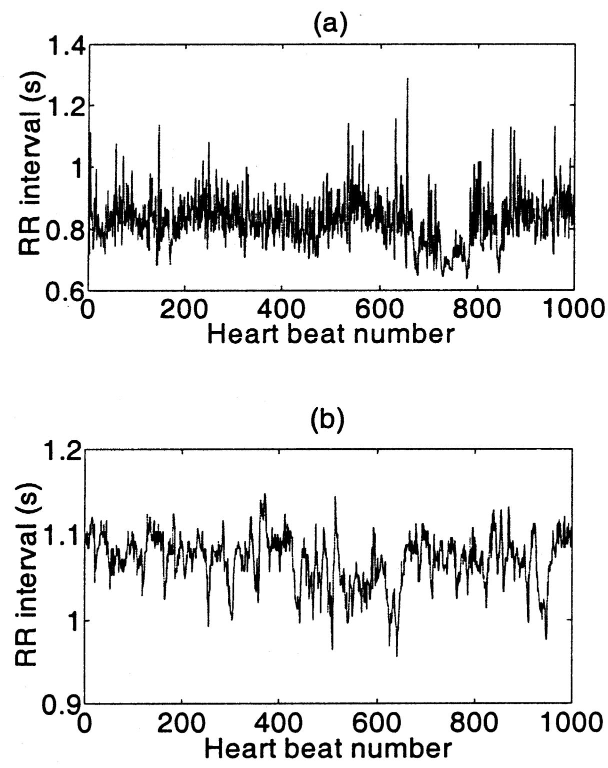

Figure 1. Typical *RR* time-series: (a) a young subject and (b) an elderly subject. A segment of 1000 *RR* intervals is shown in each case.

2.2. The wavelet transform multifractal formalism

2.2.1. Wavelet transform

The wavelet transform is obtained by projecting the signal onto a set of basis functions, obtained by dilating and translating a single prototype wavelet $\psi(t)$. Thus, the continuous wavelet transform $T_{\psi}(b, a)$ of a signal $f(t)$ is defined as

$$T_{\psi}(b, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi\left(\frac{t-b}{a}\right) dt$$

where $\psi(t)$ is the analyzing wavelet, a (> 0) is the scale parameter and b is the position parameter.

2.2.2. Analyzing wavelet

We used derivatives of the Gaussian function as analysing wavelet. The third derivative of the Gaussian function and its Fourier transform are shown in Fig. 2 at different analyzing scales.

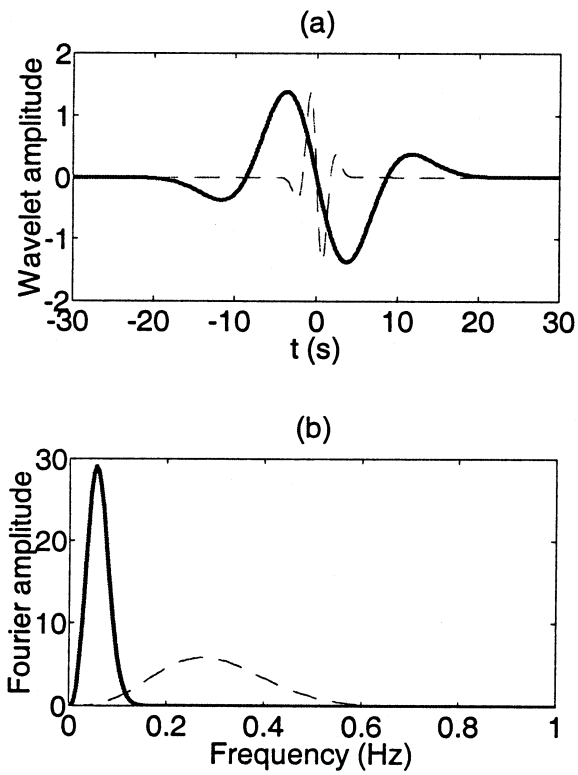

Figure 2. (a) The third derivative of the Gaussian function and (b) its Fourier transform, for $a = 1$ (- - -) and $a = 5$ (full line).

The wavelet transform thus allows good time resolution and poor frequency resolution at high frequencies, and poor time resolution and good frequency resolution at low frequencies. The wavelet transform hence entails the projection of the signal against an increasingly broader band of frequencies as the time resolution of the signal increases. A multiresolution scaling analysis of the signal is thus possible.

2.2.3. Hurst exponent

A time-dependent signal with singular or fractal features can be written as

$$f(t) = f(t_i) + (t - t_i) f'(t_i) + \frac{(t-t_i)^2}{2!} f''(t_i) +$$

$$\ldots\ldots\ldots\ldots + \frac{(t-t_i)^n}{n!} f^n(t_i) + C\left|t - t_i\right|^{h(t_i)},$$

where $h(t_i)$ is a non-integer number quantifying the local singularity of $f(t)$ at $t = t_i$ and $n < h(t_i) < n+1$. $h(t_i)$, the Hölder exponent (or the local Hurst exponent) of $f(t)$ at $t = t_i$, is extracted from the signal using the polynomial orthogonality property of wavelets [4].

2.2.4. Singularity spectrum, partition function, and multifractal formalism

Let s be a distribution and S_h the set of all the points t_i with local Hurst exponent h. The singularity spectrum $D(h)$ of s is the function which associates with any h the Hausdorff dimension, d_H, of S_h:

$$D(h) = d_H\left(\{t_i \in \Re, h(t_i) = h\}\right).$$

Also, let a partition function for any $q \in \Re$ be defined as

$$Z(q, a) = \sum_l \left(\sup_{a' \le a} \left|T_{\psi}[s](b_l(a'), a')\right|\right)^q,$$

where, for a given a, $b_l(a)$ locates the positions of the points where the wavelet transform has a local maximum modulus. These local maxima lie on maxima lines l [6]. In the limit $a \to 0^+$, it is expected that

$$Z(q, a) \sim a^{\tau(q)}.$$

The multifractal formalism relates $D(h)$ to the characteristic exponent $\tau(q)$ through the Legendre transformation:

$$D(h) = \min_q (qh - \tau(q)).$$

3. Results

3.1. Wavelet transform

The wavelet transform of an *RR* time-series is shown in Fig. 3. The analysing wavelet was the third derivative of the Gaussian function. The maxima lines are conspicuous in the associated surface plot shown in Fig. 4.

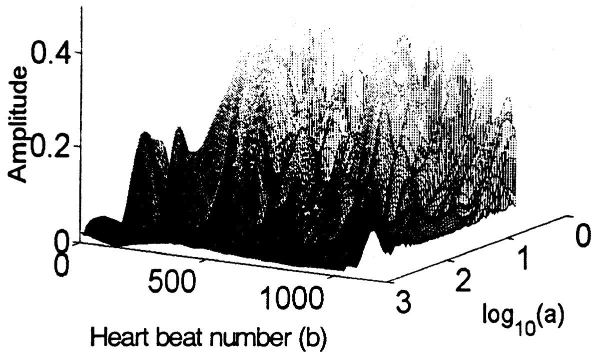

Figure 3. Wavelet transform of the *RR* interval time-series of a young subject. The analysing wavelet was the third derivative of the Gaussian function and the analyzing scale ranged from $a = 1$ to $a = 10^3$.

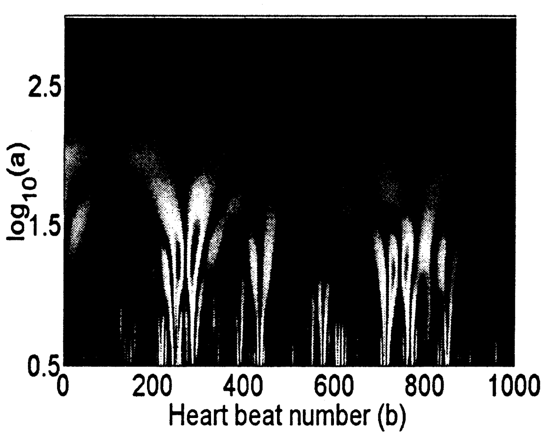

Figure 4. Surface plot associated with the wavelet transform shown in Fig. 3. The lighter shades indicate regions of higher wavelet transform amplitude.

The maxima lines are extracted from Fig. 4 to give the wavelet transform skeleton (Fig. 5).

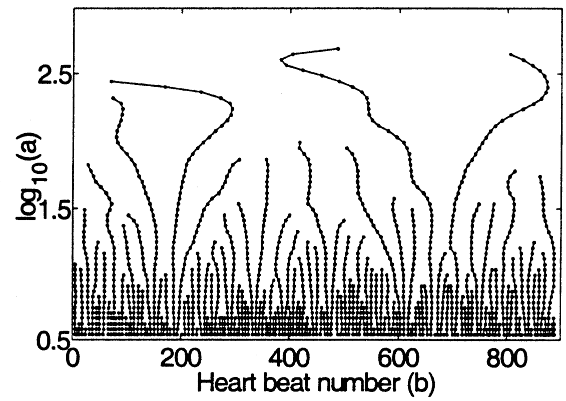

Figure 5. Wavelet transform skeleton associated with Fig. 4.

3.2. Partition function

Figure 6 shows the variation of the partition function $Z(q, a)$ for q ranging from -5 to $+5$. The scaling of the partition function with the analyzing scale a generates a characteristic exponent $\tau(q)$ from which the spectrum of singularities is computed using a Legendre transformation.

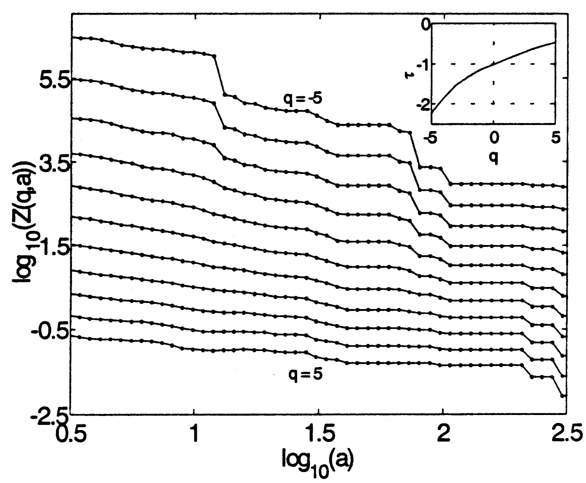

Figure 6. Scaling of the partition function $Z(q, a)$ with the analyzing scale a for a young subject. The inset shows the characteristic exponent spectrum.

3.3. Singularity spectrum

Fig. 7 shows the spectrum of singularities of a young subject. The computed results for all the 10 subjects studied are shown in Fig. 8. There is a well-defined separation between the young and the elderly group, with the spectrum of singularities peaking in the range $0.06 < h < 0.13$ for the young subjects and in the range $0.29 < h < 0.36$ for the elderly subjects.

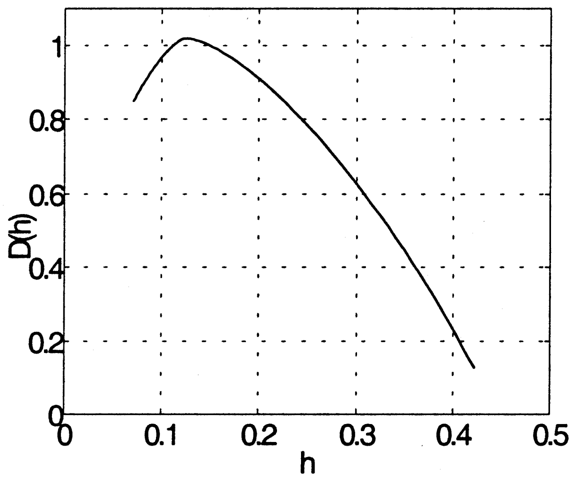

Figure 7. The spectrum of singularities of a young subject.

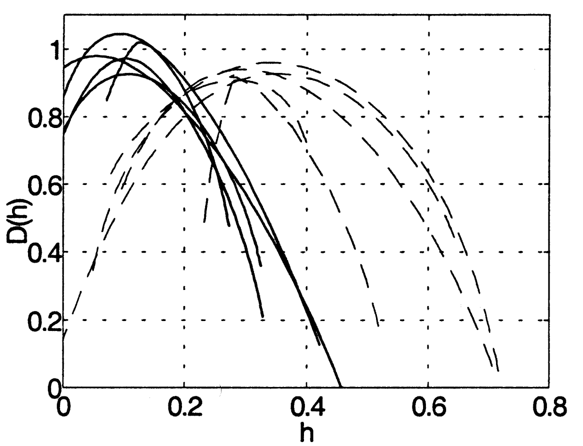

Figure 8. The spectrum of singularities for all the subjects studied. There is a well-defined separation between the young subjects (full lines) and the elderly subjects (- - -).

4. Discussion

The main objective of this study was to search for multifractal behaviour in heart rate variability using 5 young and 5 elderly subjects and to quantify the change that occurs with age from a multifractal perspective. The smooth spectrum of singularities defined over a finite range of Hurst exponents confirm the multifractal behaviour. There is also a well-defined separation between the young and the elderly group. The difference is indicative of the decrease in long-range anti-correlated behaviour in heart rate dynamics with the aging process. These results are in line with expectations that the multifractal approach can be used to robustly characterize healthy heart rate variability. However, the computations do not generate the expected sharp peaks (monofractal behaviour) in the spectrum of singularities of the elderly subjects. Sharp peaks are expected because the decrease in heart rate variability, as a result of aging, should result in a more restricted range of Hurst exponents in the spectrum of singularities. It is anticipated that the use of much larger data sets will lead to the monofractal behaviour.

Acknowledgements

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References

[1] de Gennes P-G. Scaling concepts in polymer physics. Ithaca:Cornell University Press, 1985.

[2] Halsey TC, Jensen MH, Kadanoff LP, Procaccia I, Shraiman BI. Fractal measures and their singularities: The characterisation of strange sets. Physical Review A 1986;33:1141-51.

[3] Ivanov PCh, Amaral LAN, Goldberger AL, Havlin S, Rosenblum MG, Struzik ZR, Stanley HE. Multifractality in human heartbeat dynamics. Nature 1999;399:461-5.

[4] Muzy JF, Bacry E, Arneodo A. The multifractal formalism revisited with wavelets. International Journal of Bifurcation and Chaos 1994;4:245-302.

[5] Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng CK, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals. Circulation 2000;101:e215-20.

[6] Bacry E, Muzy JF, Arneodo A. Singularity spectrum of fractal signals from wavelet analysis: Exact results. Journal of Statistical Physics 1993;70:635-74.

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