On the Presence of Deterministic Chaos in HRV Signals

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Abstract

Presence of deterministic chaos in the instantaneous heart rate variability (HRV) signal is investigated by reconstruction of a multidimensional signal using time-delay embedding method. Four methods: autocorrelation, mutual information, average displacement, and visual attractor inspection, are used to determine the time delay parameter. The dimensionality of the embedding is tested using the nearest neighbors and correlation dimension method, including surrogate data test for the correlation dimension. In addition, analysis is also performed calculating the largest Lyapunov exponents. Results obtained in the time-delay embedding process support the conclusion that the HRV series could be chaotic.

1. Introduction

The Heart Rate Variability (HRV) signal is a well known non-invasive tool for measuring the status of cardiovascular and autonomous nervous systems. Even though, there are many well established linear HRV analysis tools [1], the methods from nonlinear theory may provide more information and new diagnostic potential. The nonlinear analysis of the HRV signal is motivated by two main reasons: one is intrinsic nonlinear nature of the signal observed from a heart as a dynamical, non-linear, harmonic oscillator, whereas the other is necessity to get additional knowledge about the real phenomena. Conclusion that beat-to-beat fluctuations could be chaotic has an implication for the understanding of the nature of the autonomous nervous system and the brain. The HRV is just one observable variable from multivariable system. It is possible to reconstruct the state space of the original cardiac system, preserving the system dynamics. The most popular method for the reconstruction is the time-delay embedding. If $x(k)$ is an element of the HRV series $\boldsymbol{X}$, then a reconstructed vector $y(k)$ is introduced as:

$$y(k) = [x(k), x(k-\tau), .., x(k-(m-1)\tau)] \qquad (1)$$

where τ is chosen time delay and m is the embedding dimension (ED) [2]. The process of reconstruction is determined by a choice of the optimal parameters, ED and time delay [3].

2. Database of signals

We used seven annotated long-term recordings from the Long Term Database (ltdb) of the MIT-BIH Arrhythmia Database. These records are complete Holter tapes from seven subjects, ranging in duration from 14 to 24 hours. All signals are sampled at 128 Hz. HRV series are extracted from them, but one of the series was immediately rejected due to shortness of the data as compared to the other six series. The remaining six HRV series are "instantaneous heart rate", resampled at 2 Hz.

Table 1: Patient Data

Record	*Age*	*Sex*	*Diagnosis*
α (alpha)	46	M	ventricular tachycardia (VTA), coronary disease
γ (gamma)	47	M	arrhythmia absoluta
δ (delta)	88	M	arrhythmia and coronary disease
η (eta)	75	M	supraventricular tachycardia;coronary disease
θ (theta)	71	M	ventricular tachycardia (VTA), coronary disease
ξ (ksi)	58	M	arrhythmia absoluta after myocardial infarction

3. Methods

There is no universal method for the choice of appropriate values for the time delay and the ED, therefore we performed four different methods searching for an optimal value of τ and two methods for the choice of ED. We also applied the surrogate data test and the largest Lyapunov exponent analysis. By reconstructing and investigating the series it is possible to reach conclusions about the nature of the analyzed HRV signal.

3.1. Autocorrelation method

The autocorrelation (acf) method is the simplest method for calculating linear correlation between elements of the HRV series, for different values of τ.

$$R(\tau)=\frac{\langle x(k)\cdot x(k+\tau)\rangle_t}{\langle x(k)^2\rangle_t} \quad (2)$$

where $\langle\ \rangle$ denotes the mean. An optimal value for time delay from the acf is recommended as: the time where $R(\tau)$ reaches its first minimum or the first point of inflection of $R(\tau)$ or the time where $R(\tau)$ first drops to a certain fraction of its initial value, e.g. 1/e [4]. The acf only gives a measure of linear dependence, so it is not the most appropriate method to analyze nonlinear series.

3.2. Mutual information

The method of mutual information gives a measure of general dependence [2,4,5] between the elements of the observed series X. Mutual information M is calculated for different values of τ by:

$$M_{X,Y}(\tau,\varepsilon)=I_X(\varepsilon)+I_Y(\varepsilon)-I_{X,Y}(\tau,\varepsilon) \quad (3)$$

where an interval of values of elements $x(k)$ is divided into subintervals with length ε (one-dimensional boxes) and a scale-dependent information function is calculated:

$$I_X(\varepsilon)=-\sum_i p_1(x_i)\log_2 p_1(x_i) \quad (4)$$

where $p_1(x_i)=n_i/n$, n_i is the occupancy of the i-th box of size ε, n is the length of X. Similarly, the joint information function for series X and its delayed version Y with elements $x(k-\tau)$ dependent on τ is:

$$I_{X,Y}(\tau,\varepsilon)=-\sum_i p_2(x_i,x_{i-\tau})\log_2 p_2(x_i,x_{i-\tau}) \quad (5)$$

where $p_2(x_i,x_{i-\tau})=n_i/N$, n_i is the occupancy of the i-th two-dimensional box of linear size ε, N is the length of the two-dimensional series. For an optimal time delay it is recommended to choose the value where the function (3) reaches its first minimum. This method is only optimal for the systems having ED $m=2$ [6].

3.3. Visual attractor inspection

Visual inspection consists of plotting two-dimensional or three-dimensional projections of the reconstructed attractor for different values of τ, having delayed versions of the original HRV series on the axes of the coordinate system. In the presence of chaos, "strange attractors" show self-similarity and a feature of stretching and folding at the same time, with an exponential growth of small distances between the adjacent points on the attractor along the time. Many of them have very complicated shapes which makes it difficult to define the "strangeness" of the attractor and to choose the value of τ where it appears. Another possibility for visual inspection is to use the principal components analysis (PCA) [6]. The PCA introduces a set of orthonormal basis vectors in the embedding space such that projections onto a given number of directions preserve a maximal fraction of the variance of the original vectors. The principal directions can be obtained as the eigenvectors of the symmetric autocovarriance matrix that correspond to the largest eigenvalues. It is possible to look at the attractors having principal components on their axes, calculated for different values of τ.

3.4. Average displacement method

The average displacement method [4] is searching for an optimal time delay measuring expansion of the reconstructed attractor for different values of τ. The lag τ is chosen if the attractor is "sufficiently" expanded from the bisector line. The measure of expansion from the bisector line is defined as an average displacement:

$$S(\tau)=\frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum_{j=0}^{m-1}(x_{i+j\tau}-x_i)^2} \quad (6)$$

where m is chosen (or variable) ED and N is the length of the reconstructed time series. An empirical advise is to choose the optimal delay as a point where the slope of $S(\tau)$ first decreases to less than 40 % of its value $S(1)$, which is the main disadvantage of the method.

3.5. False nearest neighbors (FNN)

If the reconstructed attractor is projected in a lower dimension than the original, some false intersections appear and certain distant points become false neighbors [6]. If the attractor is projected in a higher dimension, some false neighbors become distant points. The procedure of increasing the embedding dimension continues either until false nearest neighbors disappear or their number reaches some acceptable level. For each element y_i of the reconstructed multidimensional series Y, according to distance $\|y_i-y_j\|$, the closest neighbor y_j is chosen. Increasing dimension i, we have:

$$R_i=\frac{|y_{i+1}-y_{j+1}|}{\|y_i-y_j\|}. \quad (7)$$

If R_i goes over some heuristic threshold, the point is considered a false nearest neighbor. For the series Y, having N points of the orbit, the percentage of false nearest neighbors of all points is calculated. The process is finished by choosing the value for ED where there are no false nearest neighbors or there is an acceptable level of them. The pollution with noise causes a permanent presence of false neighbors.

3.6. Correlation dimension

The correlation dimension is defined as:

$$\nu=d^{(2)}=\frac{1}{2-1}\lim_{\varepsilon\to 0}\frac{\log\sum_{i=1}^{N}p_i^2}{\log\varepsilon}=\lim_{\varepsilon\to 0}\frac{\log C(\varepsilon)}{\log\varepsilon}. \quad (8)$$

The correlation dimension measures spatial correlation between the points in a reconstructed multidimensional space, depending on the value of ED. The reconstructed space is divided into boxes of size ε and the probability p_i of the occupancy of the boxes is calculated [5]. Fast box counting algorithm uses specific procedure of scaling, truncating, binary conversion and interleaving. The correlation dimension is calculated for different values of reconstructing dimension ED and the optimal value of it is when ν reaches the plateau (saturation).

3.7. Largest Lyapunov exponent

The spectrum of Lyapunov exponents is a quantitative measure of evolution of adjacent phase trajectories in the reconstructed state space [7]. Positive exponents are the measure of expansion and the negative ones measure folding along the axes of reconstructed space. The presence of the largest positive Lyapunov exponent is a sign of chaotic character of data. The Wolf's algorithm observes a divergence of adjacent trajectories from the fiducial trajectory and defines the largest exponent as:

$$\lambda_1 = \frac{\sum_{i=1}^{n-1} \log_e \left(\frac{d_{i+1}}{d_i'}\right)}{t_n - t_1}. \tag{9}$$

4. Experimental results

The results for the autocorrelation method are depicted at the Fig. 1. The series θ and γ have almost monotonous decreasing trend at the lower values of τ and almost constant value for higher values of τ. The acf of series δ, ξ and η have expressed oscillatory character. The acf of α has specific oscillatory decreasing trend. The results for the application of the mutual information method show that $M(\tau)$ has trends similar to the trends of the corresponding acf. $M(\tau)$ is very low for the series γ and θ, having almost monotonous decreasing trend.

Visual analysis of the reconstructed attractors shows no decisive "strangeness" for any of the values τ. But, the PCA gives much better representation, varying values of τ between 3 and 25 and obtaining "strange" shapes of some of the attractors. After the PCA, the attractor of the series ξ has visible turbulent structure around longer axes of the spindled attractor (see Fig. 2), giving an evidence of the presence of chaos in the HRV. The attractors of γ and θ have very weak traces of a "strange" structure present (more or less) in the attractors of the other series, suggesting again a stochastic nature of those two series.

The results of the average displacement method implementation for the series γ and θ show that there is almost no expansion of the attractor for increasing values of τ and it is known as a result for stochastic signals. The results for other HRV series show "pulsating" volume of the attractors when changing τ that is especially expressed for low values of m (see Fig. 3).

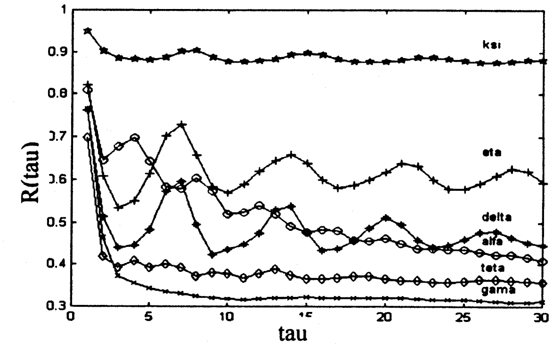

Figure 1. Autocorrelation for the HRV series (o-α, ◊-θ, *-δ, +-η, ×-γ, star-ξ)

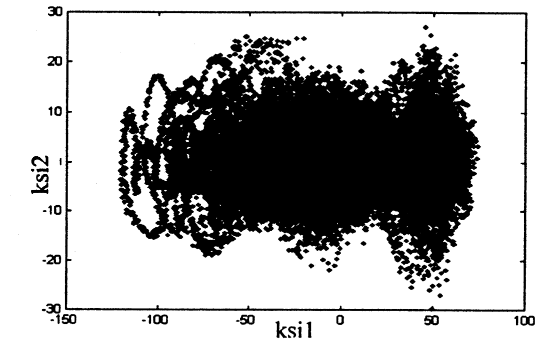

Figure 2. Projection of the attractor for the principal components 1 and 2 of the series ξ

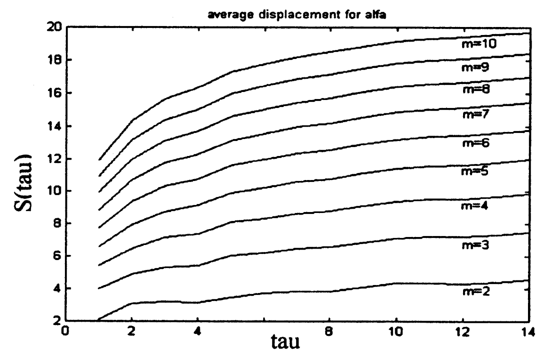

Figure 3. "Pulsating" changes of $S(\tau)$ for the series α

The FNN method gives the results as it is depicted at the Fig. 4. The percentage of false neighbors is no less than 5 % even for embedding dimension $m=20$. That could be the sign of the serious noise presence or high dimensionality of the chaotic HRV series.

The correlation dimension analysis using fast box-counting algorithm suffers from distortion (Fig. 5) due to the insufficient length of the analyzed HRV series for very high values of ED. However, the results repeat high dimensionality also obtained by the FNN method. The surrogate test gives much more objective insight into the series. For the series γ, the slope of $C(\varepsilon)$ vs. ED for the surrogate is lower than the slope of the original series. That is the sign of stochastic character. For θ the slope of

C(ε) for the original is lower than the surrogate except for lower resolution *ε*. For other series, the test of determinism is in favor of chaotic character of data.

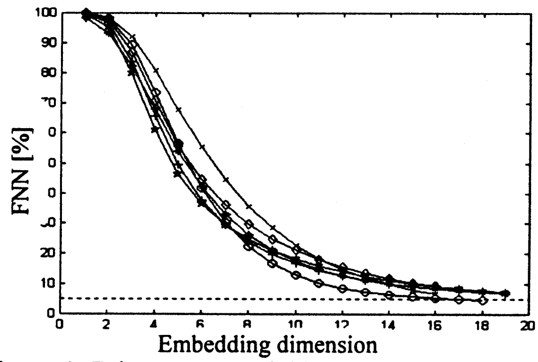

Figure 4. False nearest neighbors (FNN) for HRV series (o-α, ◊-θ, *-δ, +-η, ×-γ, ☆-ξ)

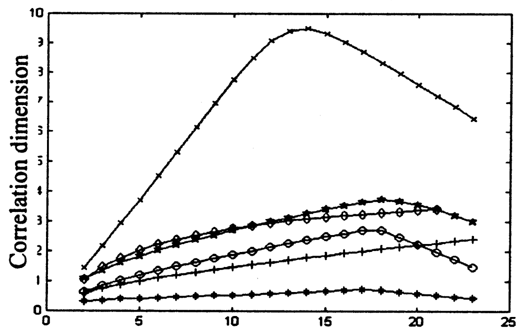

Figure 5. Correlation dimension for HRV series (o-α, ◊-θ, *-δ, +-η, ×-γ, ☆-ξ)

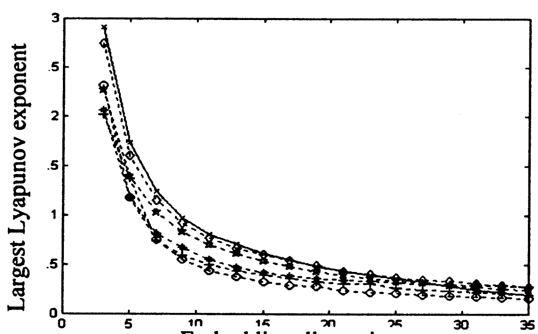

Figure 6. Largest Lyapunov exponent for HRV series (o-α, ◊-θ, *-δ, +-η, ×-γ, ☆-ξ)

The largest Lyapunov exponent is positive for all HRV series (Fig. 6.), for an interval of ED between 3 and 35, suggesting the presence of chaos. The trend of exponent calculation for γ is somewhat different having largest decreasing intention even for high ED values. Even being positive that could be a sign of infinite dimensionality. Other series also show the traces of the presence of chaos.

5. Conclusions

The obtained results indicate the presence of the deterministic chaos in four analyzed HRV series (α, δ, η, ξ). Those are: positive largest Lyapunov exponents, surrogate test for correlation dimension, shape of the attractors after the PCA, shape of the curves for the average displacement method, shapes of the acf and mutual information curves. Both methods of analysis of the ED suggest the high-dimensional chaos, with optimal choice of *τ* between 3 and 12, depending seriously on the method applied. The series γ has all those parameters (except the largest exponent) in favor of the stochastic nature, whereas θ has mixed results for different methods. The presence of the deterministic chaos in the HRV signal probably depends on the health condition of the analyzed subject, changing the character in accordance with different influences and having important implication in the analysis of organic systems.

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