Mathematical Model for Computerized Mapping of Electrocardiography: Mathematical Introduction of the Seventh Wave of the ECG

VR Abbasi

Shiraz University of Medical Sciences, Faculty of Medicine
Shiraz, Iran

Abstract

The generation and interpretation of normal and abnormal electrocardiograms through mathematical methods have been major areas of research for several decades. This paper presents a new mathematical function (Tehran-Cairo formula), which, is capable of generating normal 12-lead ECGs and patterns of abnormal electrocardiograms. In addition, the 7th wave of electrocardiography or V-wave is introduced. It has been defined mathematically and there is also, some electrophysiological evidence in favor of its existance. In this dynamic, two-dimensional model of electrocardiography, each electrocardiographic permutation of disease can be approximated through the equations. The parametric approach of the function allows replication of the electric signatures of cardiovascular disease.

1. Introduction

Automated interpretation of electrocardiograms has been put into wide use by physicians, and other health-care professionals. The generation and interpretation of normal and abnormal electrocardiograms through mathematical methods have been major areas of research for several decades and there already exists software capable of such generation and interpretation. If we consider the electrocardiograph to be a Cartesian plane, the data from each lead can be represented by a cyclic mathematical function correlating voltage to the independent variable of time: voltage=f (time). The ECG strip, then, is simply a two-dimensional plot of the heart's electrical activity along the axis of time.

This paper presents a new complex mathematical function (Tehran-Cairo formula), which, is capable of generating normal 12-lead ECGs and nearly all patterns of abnormal electrocardiograms. The feature of the formula and following calculations lead to the probable existance of the 7th wave of electrocardiography in normal and abnormal electrocardiograms particularly in presense of ST segment changes. The V-wave which is introduced for the first time in this paper has been defined mathematically and there is also, some electrophysiological evidence in favor of its existance.

2. Methods

The mathematical function most closely approximating the morphology of electrocardiograms was determined, and the coefficients refined based on the known parameters of each lead forming the 12-lead ECG. These parameters include: the width of each wave (w_n), height of each wave (h_n), rate of each wave (r_n); the phase difference of each wave relative to the second peak(R-wave) of the QRS complex ; the PR and QT intervals; For achieving a normal morphology for each lead we need a point of comparison as a criterion. In normal ECG leads it seems that the peak of R-wave is a suitable point and so, for each cycle, we compared and calculated the peak-to-peak distances and the related parameters ,which determine the phase differences between the waves to R-wave peak. However in some abnormalities like sinus arrhythmia, Type II A-V blocks and especially Wenckebach periods it is better to choose the peak of P-wave as the comparison point.

3. Results

Adopting a systematic mathematical approach to the morphology of different ECG curves in three main steps (Single wave analysis, Couple waves analysis and Final set), the final feature of Tehran-Cairo formula have been devised as follows:

$$V = p(t) + v1(t) + q(t) + r(t) + s(t) + v2\,(t) + t(t) + u(t)$$

$$V = 10.h_p\, Sin^{\left(\frac{Log\left|s_p / 10.h_p\right|}{Log\left|\cos\left(\frac{\pi.r_p.w_p}{120e}\right)\right|}\right)} \left(\frac{\pi.r_p}{60e}.t + \Phi_r + \frac{\pi.r_p\,(2\overline{PR} + w_q - w_r)}{120e}\right)$$

$$+10.h_{v1}\, Sin^{\left(\frac{Log\left|s_{v1} / 10.k_1.h_p\right|}{Log\left|\cos\left(\frac{\pi.r_p.w_{v1}}{120\,e}\right)\right|}\right)} \left(\frac{\pi.r_p}{60e}.t + \Phi_r + \Delta\Phi_{v1-r}\right)$$

$$+10.h_q\, Sin^{\left(\frac{Log\left|s_q / 10.h_q\right|}{Log\left|\cos\left(\frac{\pi.r_q.w_q}{120e}\right)\right|}\right)} \left(\frac{\pi.r_q}{60e}.t + \Phi_r + \frac{\pi.r_q\,(w_q + w_r)}{120e}\right)$$

$$+10.h_r\, Sin^{\left(\frac{Log\left|s_r / 10.h_r\right|}{Log\left|\cos\left(\frac{\pi.r_r.w_r}{120\,e}\right)\right|}\right)} \left(\frac{\pi.r_r}{60e}.t + \Phi_r\right)$$

$$+\left[10.h_s - 10..h_{v2}\, Sin^{\left(\frac{Log\left|s_{v2} / 10.h_{v2}\right|}{Log\left|\cos\left(\frac{\pi.r_t.w_{v2}}{120e}\right)\right|}\right)} \left(\frac{\pi}{2} - \Phi_s + \Phi_t + \Delta\Phi_{v2-t}\right)\right] Sin^{\left(\frac{Log\left|s_s / 10.h_s\right|}{Log\left|\cos\left(\frac{\pi.r_s.w_s}{120e}\right)\right|}\right)} \left(\frac{\pi.r_s}{60e}.t + \Phi_r - \frac{\pi.r_s\,(w_r + w_s)}{120e}\right)$$

$$+10.h_{v2}\, Sin^{\left(\frac{Log\left|s_{v2} / 10.h_{v2}\right|}{Log\left|\cos\left(\frac{\pi.r_t.w_{v2}}{120e}\right)\right|}\right)} \left(\frac{\pi.r_t}{60e}.t + \Phi_t + \Delta\Phi_{v2-t}\right)$$

$$+\left[10.h_t - 10..h_{v2}\, Sin^{\left(\frac{Log\left|s_{v2} / 10.h_{v2}\right|}{Log\left|\cos\left(\frac{\pi.r_t.w_{v2}}{120\,e}\right)\right|}\right)} \left(\frac{\pi}{2} + \Delta\Phi_{v2-t}\right)\right] Sin^{\left(\frac{Log\left|s_t / 10.h_t\right|}{Log\left|\cos\left(\frac{\pi.r_t.w_t}{120\,e}\right)\right|}\right)} \left(\frac{\pi.r_t}{60\,e}.t + \Phi_r - \frac{\pi.r_t\,(2\overline{QT} - w_q - w_r)}{120\,e}\right)$$

$$+10.h_u\, Sin^{\left(\frac{Log\left|s_u / 10.h_u\right|}{Log\left|\cos\left(\frac{\pi.r_u.w_u}{120e}\right)\right|}\right)} \left(\frac{\pi.r_u}{60e}.t + \Phi_t + \frac{\pi.r_t\,(w_t + w_u + \overline{2dTU})}{120e}\right)$$

Where: e is the spead of ECG strip; $w_p, w_q, \ldots w_u$ is the width of p,q,…,u waves in second, $r_p, r_q, \ldots, r_u$ is the rate of p,q,…,u waves in beats per minute; $h_p, h_q, \ldots h_u$ is the height of p,q,…,u waves in milivolt ; $s_p, s_q, \ldots s_u$ is the voltage of the begining and end points of of p,q,…,u waves for readers' eyes in volt to allow the reader to choose the accuracy rate , which may be decided to assign as a same value for all waves ; PR,QT and dTU is the length of PR and QT interval and the distance between the ending point of T-wave and the beginning point of U-wave, respectively and Φr is the optional distance between the first peak of R-wave and the point of reference on the time axis; and Φs and Φt are the phase difference of S and T-wave with the peak of R-wave.

The coefficients of the formula for a typical 12-lead ECG of a 22 years old man were calculated. For evaluating these coefficients the parameters or properties of the waves of each lead - obtained manually was substituted in the formula and then the 32 simplified coefficients for each lead are found. These coefficients are displayed in a 12.32 matrix.

That coefficients of the Tehran-Cairo formula should be defined in a range. To determine these ranges we need a statistical population study, considering height, weight, age and sex, which requires further studies. A similar, but larger, matrix has been partially completed representing patterns of electrical abnormality common to some cardiac diseases.

The V-wave is a result of the Tehran-Cairo formula and the following calculations in both normal and abnormal electrocardiograms. It is a biphasic hidden wave, which is usually located behind the PR segment, QRS complex and the ST segment. Normally it generates downward slope of PR segment and upward slope of ST segment. This wave is also, very important in generating the graphs of myocardial ischemia and other ST segment changes.

V-wave (V_1 & V_2 parts) properties should be defined with parameters of other waves. The properties needs some empirical mathematical criteria, which have already been tested successfully for generating normal 12-leads ECG, although they may require some changes in later studies. These criteria are suggested as below:

$h_{v1} = k_1 . h_p$ ($-0.45 < k_1 < -0.35$ in limb leads and $-0.3 < k_1 < -0.2$ in precordial leads)

$w_{v1} = \overline{PR} - w_p + w_q + \frac{1}{2} . w_r$ (for all leads)

$\Delta\Phi_{v1-r}$ $\frac{\pi . r_r (w_q + w_r)}{60e}$ (The phase difference between V_2 and R-wave for all leads)

The axis of V_2–wave seems to have a smaller angle with T-wave rather than the QRS complex. Therefore, to achieve simple linear equations for V_2-wave definition and reduce the required spatial correction to the least value, It is better compared with T-wave; So in all leads we have:

$$h_{v2} = \frac{h_t}{k_2} \quad (2.5 < k_2 < 4.4, \overline{k_2} = 3.22)$$

$$w_{v2} = k_3 . w_t \quad (1.25 < k_3 < 1.45, \overline{k_3} = 1.40)$$

$$\Delta\Phi_{v2-t} = \frac{\pi . r_t . k_4 (2\overline{QT} - 2w_q - w_R - w_t)}{120e}$$

$$(0.22 < k_4 < 0.40, \overline{k_4} = 0.313)$$

4. Conclusion

The main feature of the formula contains properties of ECG waves including width, height and rate. In many of ECG abnormalities like arrhythmias and chamber hypertrophies, these three principal properties change frequently. Different rates and abnormal phase difference between ECG waves are seen in many kinds of arrhythmias.

Twin waves (like RR' and SS' complexes in bundle branch blocks and all assymmetrical waves) that are defined as two mixed waves, with a peak-to-peak distance less than half of the summation of their widths. These combinations should also be corrected with parameters like peak-to-peak distance of two mixed waves, slope percentage in each side of asymmetrical wave and the location of meplat points.

The basic function is flexible enough to incorporate multiple abnormalities in a single electrocardiogram. The efficacy and accuracy of this system in the clinical settings still requires much more research and testing to reliably serve our healthcare concern.

Acknowledgments

I have warm thanks for Dr. Shahdad Khosropanah, Dr. Mohammad A. Ostovan, Dr. Karim Mortazavi, Dr. Ali Reza Tehrani and Dr. Barmak Yaghoobian in Shiraz University of Medical Sciences, for their helpful discussions, Mr Ramin Raoof for his editorial help and I will always have very special thanks and regards to my dear father, Mohammad H. Abbasi, Mechanical engineer, who made me interested in physics and Mathematics with spending so many hours.

References

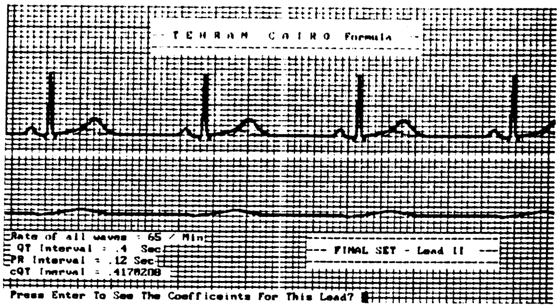
[1] Leo Schamroth: An Introduction to Electrocardiography. Ontario, The C.V. Mosby Company. 1990

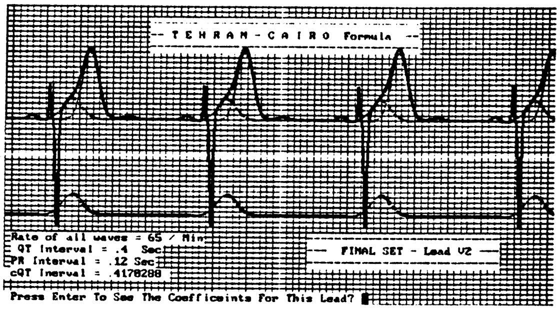
[2] Marvin I. Dunn , Bernard S. Lipman: Lipman-Massie Clinical Electrocardiography. Chicago, Year book medical publisher, Inc.1989

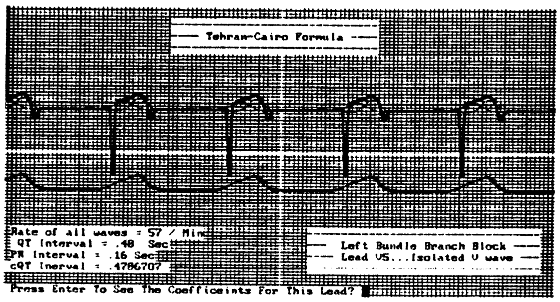
[3] Don Inman , Bob Albercht : Qbasic Made Easy. Newyork, Osborn McGraw-Hill Book Co. 1991

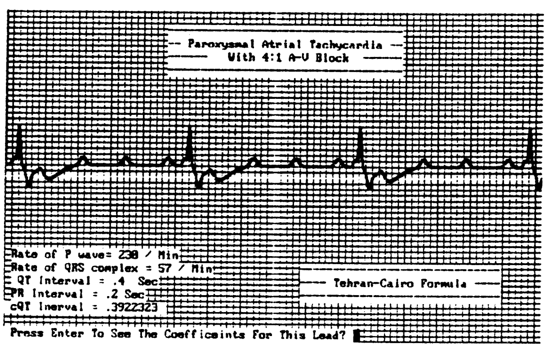
[4] David G. Crowdis, Susanne M. Shelley, Brandon W. Wheeler : Calculus for business, Biology, and The social Sciences. London, Collier Macmillan Publishers. 1976

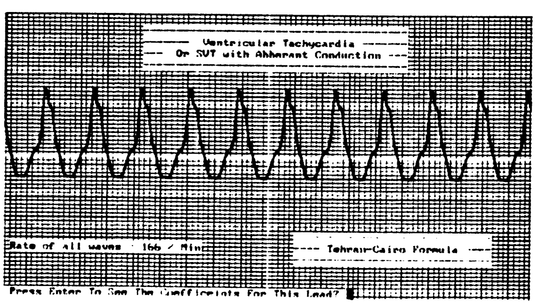
Address for correspondence. E-mail: abbasiv@sums.ac.ir

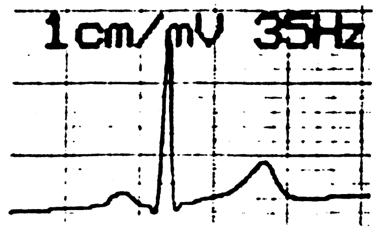

-- T E H R A N C A I R O Formula --
Rate of all waves : 65 / Min
QT Interval : .4 Sec
PR Interval = .12 Sec
cQT Inerval = .4170288
--- FINAL SET - Lead II ---
Press Enter To See The Coefficeints For This Lead?

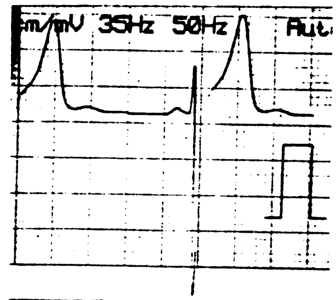

-- T E H R A N - C A I R O Formula --
Rate of all waves = 65 / Min
QT Interval = .4 Sec
PR Interval = .12 Sec
cQT Inerval = .4170288
--- FINAL SET - Lead V2 ---
Press Enter To See The Coefficeints For This Lead?


---- Tehran-Cairo Formula ----
Rate of all waves = 57 / Min
QT Interval = .48 Sec
PR Interval = .16 Sec
cQT Inerval = .4706707
---- Left Bundle Branch Block ----
--- Lead V5....Isolated V wave ---
Press Enter To See The Coefficeints For This Lead?


-- Paroxysmal Atrial Tachycardia --
------ With 4:1 A-V Block ------
Rate of P wave= 238 / Min
Rate of QRS complex = 57 / Min
QT Interval = .4 Sec
PR Interval = .2 Sec
cQT Inerval = .3922323
---- Tehran-Cairo Formula ----
Press Enter To See The Coefficeints For This Lead?


-------- Ventricular Tachycardia --------
--- Or SVT with Abberant Conduction ---
Rate of all waves : 166 / Min
---- Tehran-Cairo Formula ----


1cm/mV 35Hz


m/mV 35Hz 50Hz