A Comparison of Zero and First Order Tikhonov Regularization for an Inhomogeneous Volume

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Abstract

In this study we examine the effects of tissue inhomogeneities on epicardial estimates when using zero order Tikhonov and first order Tikhonov regularization. Epicardial potentials measured during an in-vivo swine experiment were projected to 96 body surface locations using an inhomogeneous transfer matrix constructed using a finite element model of the swine made from CT scans. It was assumed that the lungs, fat, and remaining volume had conductivities of 50, 40, and 220 μmho/mm. The inverse estimates were then computed from the body surface potentials where noise had been added to simulate measurement errors. The inverses were computed using the same inhomogeneous transfer matrix used for the the forward projection, and with a homogeneous transfer matrix.

1. Introduction

In estimating epicardial potentials from measured body surface potentials, the inverse problem of electrocardiography, it is usually necessary to construct a transfer matrix relating the epicardial potentials to the body surface potentials (or Laplacians). In addition to incorporating geometrical information, the transfer matrix must incorporate at least some of the tissue electrical properties. In an effort to construct this matrix more rapidly, it would be beneficial to know to what extent the electrical properties of the tissues between the epicardium and the body surface (i.e., lungs, fat, etc.) need to be included. It has been argued that these inhomogeneities only lead to amplitude changes in the estimated epicardial potentials, but that the general shape of the estimated epicardial potentials remains the same [1].

In this study, we compared the errors between estimated epicardial potentials when an inhomogeneous transfer matrix was used for the inverse and when a homogeneous transfer matrix was used for the inverse. For the purpose of simplicity in this study, we examined the two commonly used inverse techniques, zero and first order Tikhonov regularization [2, 3]. The errors between estimated and true epicardial potentials were measured using the relative error, which penalizes amplitude differences between signals, and the correlation coefficient, which does not penalize amplitude differences.

2. Experimental data and model

For this paper, data was collected during an *in-vivo* experiment on swine. The swine model was used because of its similarity to humans in the anatomical arrangement of heart, lungs, bone, muscle, etc. For this experiment, bipolar pacing electrodes were sewn to the heart surface (the epicardium) in six different locations, and an epicardial sock was placed over the heart surface and over the pacing electrodes. The epicardial sock had effectively nine columns of six electrodes arranged about the heart. The chest was then sewn shut and unipolar recordings of epicardial potentials were made from the epicardial sock electrodes while the heart was paced from the six sites (Prucka Engineering, Houston, Texas).

A finite element model of the region from the epicardium to the torso was constructed using the I-DEAS finite element package (SDRC,Ohio) from CT scans made of the swine. For this study we assumed either an inhomogeneous or a homogeneous model with 41,430 nodes and 212,366 linear tetrahedral elements. For the inhomogeneous transfer matrix, it was assumed the lungs, fat, and remaining volume had conductivities of 50, 40, and 220 micro mho/mm. The finite element meshes were the same for both transfer matrices, only the assumed conductivities changed. The finite element model had 1748 nodes on the epicardium. In order to project the measured epicardial potentials at the 54 electrodes to the torso, we performed Laplacian interpolation from the measured electrodes to estimate the potentials at the remaining finite element nodes on the heart surface [4]. Using standard

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finite element techniques [2] a transfer matrix relating the (measured and estimated) epicardial potentials at all nodes on the heart surface to finite element nodes on the body surface was constructed. Specifically, the inhomogeneous transfer matrix was used in the forward problem to relate the potentials at all 1748 epicardial nodes to 96 finite element nodes on the torso (body) surface which we assumed to be the measurement locations. These 96 torso surface locations were fairly evenly spaced near the heart on the pig torso.

The time segments analyzed consisted of the QRS portion of the cardiac cycle beginning just after the pacing spike. The duration of the QRS varies as the source of the ventricular depolarization is varied, from a minimum of 75 milliseconds to 120 milliseconds.

In order to simulate modeling and measurement errors, white Gaussian noise was added to the computed body surface potentials. Once the body surface potentials were computed, the segments of time to be analyzed were determined, and the corresponding body surface potentials were determined. The rms value of the body surface potentials over the entire time period to be analyzed was determined as

$$RMS = \sqrt{\frac{1}{96N}\sum_{k=1}^{96}\sum_{i=1}^{N}(b_k^i)^2} \quad (1)$$

where N is the number of sample points in the QRS to be analyzed, b_k^i is the i^{th} sample point in the QRS at the k^{th} location. Once the RMS value was determined, zero mean white Gaussian noise with standard deviations of

$$\sigma = RMS * f \quad (2)$$

was added to the body surface potentials, where f was varied. Specifically, f was 0.05 for the 5% noise level used throughout this paper, although other levels of noise were also examined.

3. Inverse algorithms

Both zero and first order Tikhonov regularization can be formulated as solutions to the following minimization problem

$$\min_{\hat{\underline{h}}} \quad \Pi = ||\hat{\underline{b}} - \underline{b}||^2 + t||\mathbf{R}\hat{\underline{h}}||^2 \quad (3)$$

where $\underline{b}$ is a vector of known (measured) body surface potentials, $\hat{\underline{h}}$ and $\hat{\underline{b}}$ are estimates of the potentials on the heart surface (the *epicardium*) and body surface, respectively, $\mathbf{R}$ is the (matrix) regularization operator, and t is the regularization parameter. In general, we attempt to match the estimated body surface potentials with the measured body surface potentials, while penalizing epicardial estimates with large magnitude. The regularization parameter t indicates the relative weight given to the two terms, and needs to be estimated based on measurable data. For zero order regularization the matrix operator $\mathbf{R}$ is the identity matrix and the amplitude of the epicardial estimates are penalized. For first order regularization the matrix operator $\mathbf{R}$ is a surface gradient operator and the first derivatives of the epicardial estimates are penalized. The estimated body surface and epicardial potentials are related through the transfer matrix,

$$\hat{\underline{b}} = \mathbf{Z}\hat{\underline{h}} \quad (4)$$

Here $\mathbf{Z}$ could be either the inhomogeneous transfer matrix or the homogeneous transfer matrix. The regularization parameter t was chosen by the composite residual and smoothing operator (CRESO). The performance of the two algorithms was measured using the correlation coefficient between the true and estimated epicardial electrograms, since the correlation coefficient was independent of the amplitude differences.

4. Results and discussion

Figures 1 and 2 present the results for twenty-five simulations for the six protocols. Figure 1 displays the relative errors (mean ± standard deviation) between the estimated epicardial potentials and the true epicardial potentials using the homogeneous and inhomogeneous transfer matrices with both zero and first order Tikhnov regularization. Figure 2 displays the correlation coefficients (mean ± standard deviation) between the estimated epicardial potentials and the true epicardial potentials using the homogeneous and inhomogeneous transfer matrices with both zero and first order Tikhonov regularization.

Figure 1 indicates, that the use of first order Tikhonov regularization consistently produces smaller relative errors than does zero order Tikhonov regularization, whether the homogeneous or inhomogeneous transfer matrix is used to compute the epicardial estimate. It also appears, however, that first order Tikhonov regularization, which penalizes the first derivative of the estimate, is much more affected by the use of the homogeneous transfer matrix to compute the inverse than is zero order Tikhonov regularization, which penalizes the amplitude of the estimated epicardial potentials. In Figure 2 we use the correlation coefficient to determine the match between the estimated and measured epicardial potentials. Again, as with the relative error measure, the correlation coefficients between the estimated and measured epicardial potentials are consistently higher for first order Tikhonov regularization than for zero order Tikhonov regularization,

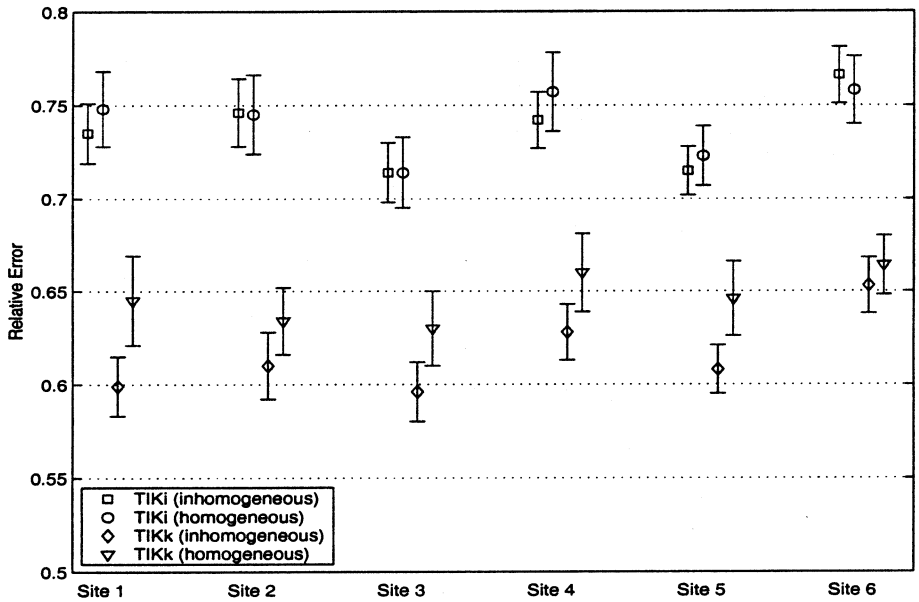

Figure 1. Relative errors (mean ± standard deviations for 25 simulations) for epicardial estimates computed from zero order Tikhonov regularization (TIKi) and first order Tikhonov regularization (TIKk) for the six pacing sites. The relative error is not independent of amplitude variations between the estimated and true (measured) epicardial signals.

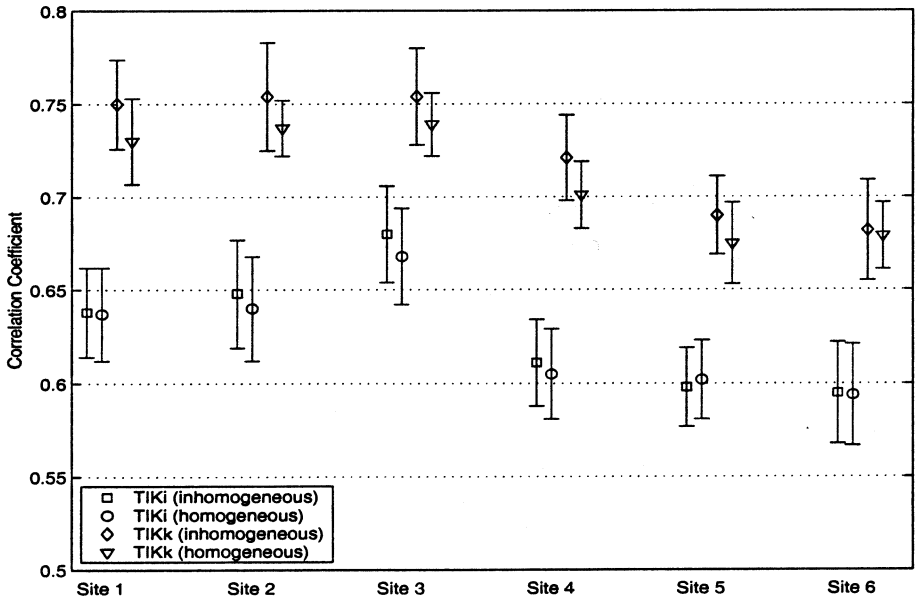

Figure 2. Correlation coefficients (mean ± standard deviations for 25 simulations) for epicardial estimates computed from zero order Tikhonov regularization (TIKi) and first order Tikhonov regularization (TIKk) for the six pacing sites. The correlation coefficient is independent of amplitude variations between the estimated and true (measured) epicardial signals.

whether or not the inhomogeneous transfer matrix is used to compute the inverse.

Since the correlation coefficient is independent of amplitude differences between the estimated epicardial potentials and the measured potentials, if the only differences between estimates using the homogeneous transfer matrix and the inhomogeneous transfer matrix was the amplitude of the estimated potentials, the correlation coefficients for the two cases should be the same. This is very nearly true for the case of zero order Tikhonov regularization. However, for first order regularization, it appears that there is more than just an amplitude change when the different transfer matrices are used.

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