

# Automating the Selection of Expansion Modes Using the Principal Components of Time Method for Solving the Inverse Problem of Electrocardiography

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## Abstract

*A common approach for estimating epicardial potentials from measured body surface potentials is to solve the problem using some type of regularization at each point in time independently. Greensite and Huiskamp have previously proposed a solution based on expanding the solution in the principle time components of the data. However, their proposed method did not indicate how many modes to use in this expansion, or indicate how to automate it. In this paper, we examined an automated solution to this problem. This automated procedure was applied to data obtained during a study on swine. For first order Tikhonov regularization, the new method produced an average relative error of 0.56 over all six depolarization sequences, compared with 0.61 for the traditional approach. While the new automated method is not necessarily optimal, it does consistently produce smaller relative errors than the traditional approach.*

## 1. Introduction

A common approach for estimating epicardial potentials from measured body surface potentials is to solve the problem using some type of regularization at each point in time independently, with a different regularization parameter at each time sample. In a previous paper, we illustrated that postprocessing these electrograms with a simple moving average filter often produced more accurate estimates of epicardial electrograms [1].

There have been a number of approaches which attempt to combine temporal spatial information directly in the problem formulation, so that any epicardial estimates produced by these methods will inherently include this information [2-5]. One such method is that of Greensite and Huiskamp [6] who proposed expanding the solution in the principle time components of the data. However, their proposed method did not indicate how many modes to use

in this expansion, or indicate how to automate it. In this paper, we examined an automated solution to this problem.

## 2. Methods

*Algorithms.* Assume the measured body surface potentials at time  $i$  are denoted by  $\underline{b}_i$ , the epicardial potentials at the same time instant are denoted  $\underline{e}_i$ , and the transfer matrix relating the epicardial potentials to the body surface potentials is given by  $Z$ , so we have

$$\underline{b}_i = Z \underline{e}_i \quad (1)$$

Often this equation is solved for  $\underline{e}_i$  at each sample time independently using either truncated singular value decomposition or some form of Tikhonov regularization. Denote the singular value decomposition of the transfer matrix  $Z$  by

$$Z = U_z \Sigma_z V_z^T \quad (2)$$

One alternative, proposed by Greensite and Huiskamp, is to determine the principle components in time of the measured body surface potentials and only use the dominant principle components.

First, all the measured body surface potentials from time index 1 to  $N$  are put into one array

$$B = [\underline{b}_1 \ \underline{b}_2 \ \dots \ \underline{b}_N] \quad (3)$$

The singular value decomposition of  $B$  is denoted as

$$B = U_b \Sigma_b V_b^T \quad (4)$$

Note that the columns of  $V_b$  give the principle components of the (time domain) measured data. The basic idea is to limit the number of principle components we use in the estimated epicardial potentials. The epicardial potentials  $E$  for time index 1 to  $N$  are then related to the measured body surface potentials  $B$  by

$$B = Z E \quad (5)$$

or

$$U_b \Sigma_b V_b^T = ZE \quad (6)$$

$$U_b \Sigma_b = ZEV_b \quad (7)$$

$$U_b \Sigma_b = Z\gamma \quad (8)$$

where

$$\gamma = EV_b \quad (9)$$

Hence if we can solve for  $\gamma$  we can compute the estimated epicardial potentials as

$$E = \gamma V_b^T \quad (10)$$

We can solve the inverse problem represented by the equation

$$U_b \Sigma_b = Z\gamma \quad (11)$$

using any standard method for solving inverse problems. In this paper we used first order Tikhonov regularization. Since our goal is to limit the expansion to the principle components in the time direction, we wish to solve the system of equations

$$U_{b,i} \Sigma_{b,i} = Z\gamma_i \quad (12)$$

where  $U_{b,i}$  is the  $i^{th}$  column of  $U_b$ ,  $\Sigma_{b,i}$  is the  $i^{th}$  singular value, and  $\gamma_i$  is the  $i^{th}$  expansion vector for the epicardial potentials. We then form  $\gamma^*$  as

$$\gamma^* = [\gamma_1 \ \gamma_2 \ \dots \ \gamma_k] \quad (13)$$

Denote  $V_b^*$  as the first  $k$  rows (first  $k$  principle components) of  $V_b^T$ . Then our optimal estimate of the epicardial potentials  $E^*$  is given by

$$E^* = \gamma^* V_b^* \quad (14)$$

The only remaining task is to determine the number of principle time expansion modes  $k$ , and the number of modes (truncation rank) to used in solving 12 for each principle time component  $i$ .

We only solve the  $k^{th}$  principle component time equation if the discrete Picard condition is satisfied, that is, if the initial part of the sequence  $\langle U_{b,k}, V_{z,n} \rangle^2$  decrease on average faster than the square of the singular values  $\Sigma_{b,n}^2$  as  $n$  increases. In this study we only examined at most eight principle mode time equations ( $k \leq 8$ ).

To determine the number of expansion modes to use for the  $k^{th}$  principle time component, we utilize the following algorithm:

1. determine the sequence  $\langle U_{b,k}, V_{z,n} \rangle^2$
2. determine the square of the singular values  $\Sigma_{b,n}^2$

3. fit the sequences from step 1 and step 2 with a third order polynomial

4. obtain the first derivative of these polynomials

5. equate the two derivatives and solve to determine the truncation rank:

(a) If one solution exists take it to be the truncation rank

(b) If more than one solution exists take the maximum solution to be the truncation rank for the first four principle time components, otherwise take the minimum solution to be the truncation rank

(c) If no solution exists, do not use this or any higher principle time component in the solution.

Once the number of principle components and the truncation rank was determined, each principle time equation was solved using first order Tikhonov regularization with CRESO to choose the regularization operator.

*Data.* Data was collected during an *in-vivo* experiment on swine. Bipolar pacing electrodes were sewn to the heart surface (the epicardium) in six different locations, and an epicardial sock was placed over the heart surface and over the pacing electrodes. The epicardial sock had effectively nine columns of six electrodes arranged about the heart. The chest was then sewn shut and unipolar recordings of epicardial potentials were made from the epicardial sock electrodes while the heart was paced from six sites.

A finite element model of the region from the epicardium to the torso was constructed using the I-DEAS finite element package (SDRC, Ohio) from CT scans made of the swine. Our finite element model included 41,430 nodes and 212,366 linear tetrahedral elements, and had 1748 nodes on the epicardium. In order to project the measured epicardial potentials at the 54 electrodes to the torso, we performed Laplacian interpolation from the measured electrodes to estimate the potentials at the remaining finite element nodes on the heart surface. Using standard finite element techniques a transfer matrix was constructed relating the (measured and estimated) epicardial potentials at all nodes on the heart surface to finite element nodes on the body surface. Specifically, our transfer matrix related the potentials at all 1748 epicardial nodes to 96 finite element nodes on the torso (body) surface which we assumed to be the measurement locations. These 96 torso surface locations were fairly evenly spaced near the heart on the pig torso.

In order to simulate modeling and measurement errors, zero mean white Gaussian noise with a standard deviation given by

$$\sigma = f * rms \quad (15)$$

where  $f$  is 0.05 for a 5% noise level, was added to the forward projected body surface data. Twenty five simulations were performed for each pacing site.

### 3. Results

Figure 1 displays the sequences  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  and the third degree polynomials fit to these points as a function of expansion mode  $n$  for the first principle time mode for pacing protocol 6. Figure 2 displays the estimated slopes for the sequences displayed in Figure 1. Hence for the first principle time expansion mode we should use 32 modes in the solution.

Figure 3 displays the both the sequences  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  and the third degree polynomials fit to these points as a function of expansion mode  $n$  for the fourth principle time mode for pacing protocol 6. Figure 4 displays the estimated slopes for the sequences displayed in Figure 3. Hence for the first principle time expansion mode we should use 27 modes in the solution.

Table 1 summarizes the results using the above algorithm to determine the number of principle time components and expansion modes to solve equation 12 using first order Tikhonov regularization. The results in the table are the average relative errors between the 54 predicted electrograms and estimated electrograms averaged over 25 simulations for the six pacing sites.

Table 1. Average relative errors for the six pacing sites using first order Tikhonov regularization.

Pacing Site	Time Independent	Principle Time
1	0.60	0.55
2	0.60	0.56
3	0.52	0.48
4	0.71	0.66
5	0.54	0.49
6	0.68	0.62

### 4. Discussion and conclusions

Utilizing an expansion based on a limited number of principle time components combined with our new automated algorithm yielded estimated electrograms with

smaller average relative errors than electrograms estimated at each time step independently for first order Tikhonov regularization. The results utilizing the new algorithm were consistently better for all pacing sites (depolarization sequences). However, the algorithm developed is not yet optimized and it is still probably possible to obtain better results.

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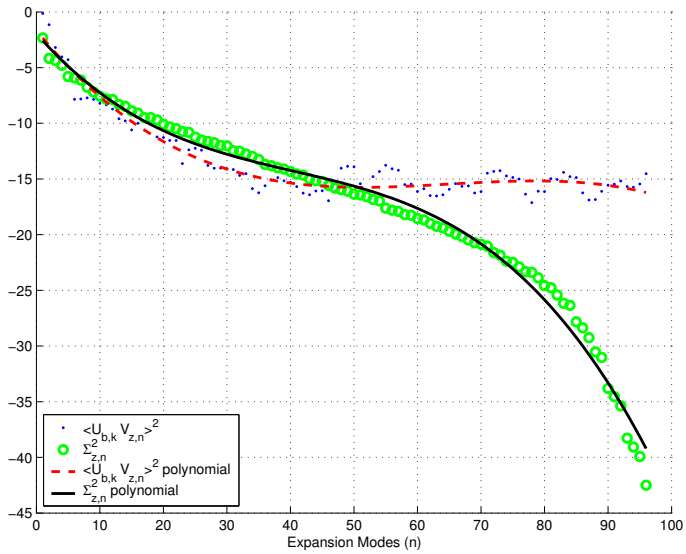


Figure 1. The sequences  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  and the corresponding third degree polynomials fit to these points as a function of expansion mode  $n$  for the first ( $k = 1$ ) principle time mode. The ordinate is a logarithmic scale.

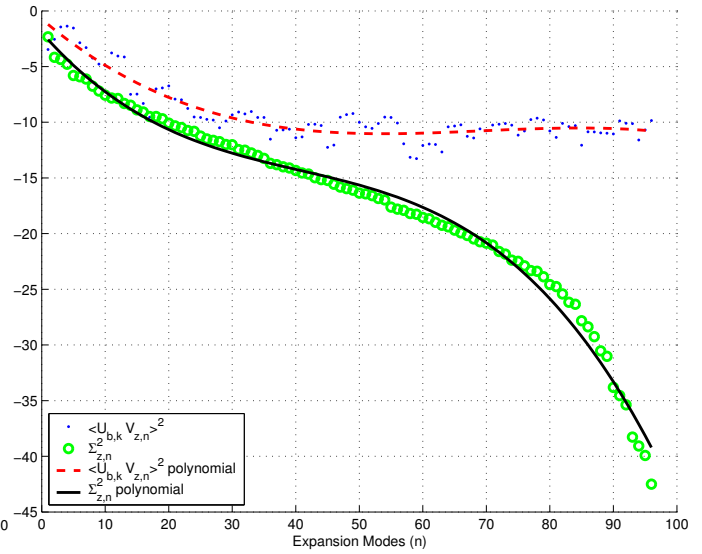


Figure 3. The sequences  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  and the corresponding third degree polynomials fit to these points as a function of expansion mode  $n$  for the fourth ( $k = 4$ ) principle time mode. The ordinate is a logarithmic scale.

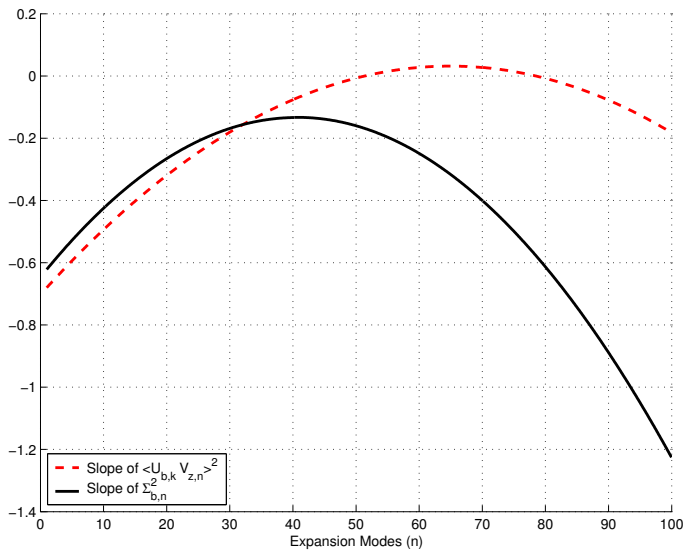


Figure 2. The estimated slopes of  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  as a function of expansion mode  $n$  for the first ( $k = 1$ ) principle time mode. The ordinate is a logarithmic scale.

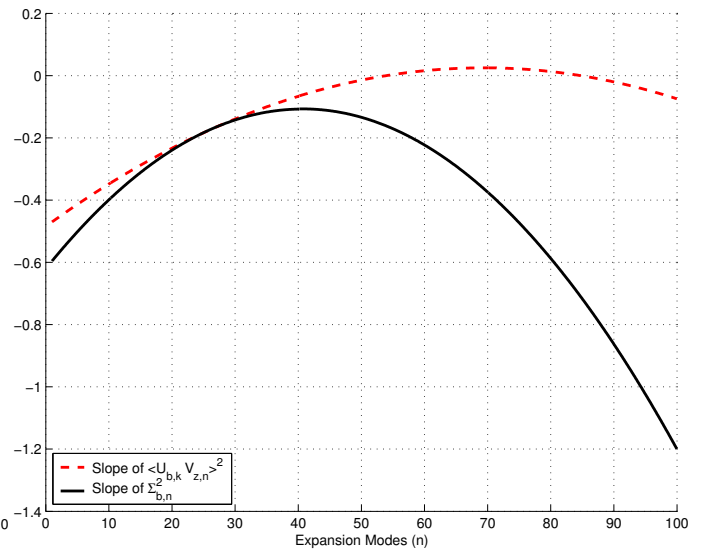


Figure 4. The estimated slopes of  $\langle U_{b,k}, V_{z,n} \rangle^2$  and  $\Sigma_{b,n}^2$  as a function of expansion mode  $n$  for the fourth ( $k = 4$ ) principle time mode. The ordinate is a logarithmic scale.