The First Order Absolute Central Moment as an Edge-Detector in Cardiovascular Imaging: A Comparison with Two Well-Known Edge-Detectors

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Abstract

Cardiovascular imaging usually requires the detection and the localization of contours. Many mathematical operators have been studied to improve the performances of the edge detection algorithms but the most frequently used operators in literature remain the Laplacian of Gaussian (LoG) and the gradient of Gaussian (GoG). Recently, a new mathematical operator, which has been obtained from the generalization of the first absolute central moment, has been proposed. The aim of this paper is to compare the edge detection and localization capabilities of this operator with those of LoG and GoG.

1. Introduction

Gray-level moments must not be confused with spatial moments. A brief description of the properties of these two different kinds of moments and of their use in image processing systems can be found in [1]. The spatial moments are often called shape moments or geometric moments since they provide us with information on the geometric features of an object. In image processing spatial moments are commonly used to identify visual patterns independently of their position, orientation and size. The moment representation theorem guarantees that a density distribution $f(x,y)$, which is bounded and defined on a finite domain, can be uniquely determined by its moments. Such moments are often used in character recognition.

Unlike spatial moments, gray-level central moments or intensity moments provide us with information on the distribution of the gray-levels which belong to a region of the image. Conceptually, they are dispersion indexes and are used in image processing literature to describe how the gray-levels of a finite domain Ω of the image are distributed with respect to the mean level computed on Ω itself. It is worth noting that, since these moments do not take into account the spatial relationships among the gray-levels of Ω, their values remain unchanged under an arbitrary spatial rearrangement of the pixels. Therefore, the gray-level central moments are mostly used to describe the features of the histogram computed on Ω. In order to describe more complex image features such as texture, higher order statistics of the gray levels must be used. For example, a more useful description of textures is obtained if the gray-level cooccurrence matrix is used [1].

However, the gray-level central moments have other properties that have never been investigated in depth in the past. These moments can accomplish a lot more than simply describing the features of a histogram. They can also be used to detect and locate edges as well as locating lines and highlighting key points such as corners and junctions. In [2] a filter derived from the generalization of the first absolute central moment is used as the first stage of an automatic contour tracking procedure. The paper shows how localization failures at corners and junctions can be overcome by substituting the generalized first absolute central moment for the conventional GoG magnitude. In [3] the mass center of the gray-level variability, associated to the first absolute central moment, is also defined and it is shown how this new operator can be exploited to locate edges. In this paper, the edge detection and localization capabilities of the first absolute central moment are compared with those of other standard operators commonly used in cardiovascular image processing. I particular the GoG filter, the Canny filter and the Marr-Hildreth procedure based on the LoG filter are considered.

2. Edge-detectors

Let $f(x,y)$ be the gray level map of an image, and let $g(x,y,\sigma_i)$ be Gaussian weight functions with a unitary integral over a circular domain Ω with aperture $r=3\sigma_i$. The following relationship is used to compute the first absolute central moment at a point $\mathbf{p}\equiv(x,y)$

$$e(\mathbf{p}) = \iint_{\Omega} \left| f(\mathbf{p}-\boldsymbol{\tau}) - f(\mathbf{p}) \otimes g(\mathbf{p},\sigma_1) \right| g(\boldsymbol{\tau},\sigma_2) d\tau_x d\tau_y \qquad (1)$$

where the symbol $\otimes$ represents the convolution operator. Gaussian functions are used since the Gaussian has many qualities which make this function a unique operator in early image processing. Eq.(1) highlights gray level discontinuities with a ridge having a bell profile and the top of the ridge locates the points of the discontinuity. The mass center **b** of the gray level variability associated to the first absolute central moment can be also defined and computed at a point **p**:

$$\mathbf{b}(\mathbf{p}) = \begin{cases} \frac{1}{e(\mathbf{p})} \iint_{\Omega} \left| f(\mathbf{p}-\boldsymbol{\tau}) - f(\mathbf{p}) \otimes g(\mathbf{p},\sigma_1) \right| \boldsymbol{\tau} g(\tau,\sigma_2) d\tau_x d\tau_y & e(\mathbf{p}) \neq 0 \\ 0 & e(\mathbf{p}) = 0 \end{cases} \qquad (2)$$

In [3] it is shown that vector $\mathbf{b}(\mathbf{p})$ always indicates the direction of the path that joins point **p** to the nearest point of the nearest discontinuity. Moreover, when configurations with $\sigma_1 > \sigma_2/\pi$ are chosen, the mass center of the gray level variability is always closer to the discontinuity than point **p**. That is, the center of mass of the gray level variability can approach the discontinuity independently of the distance between the starting point and the discontinuity. This is an important property which allows us to localize a gray level discontinuity with an iterative approach.

The GoG magnitude is defined as

$$h(\mathbf{p}) = \sqrt{(f(\mathbf{p}) \otimes g_x(\mathbf{p}))^2 + (f(\mathbf{p}) \otimes g_y(\mathbf{p}))^2} \qquad (3)$$

where $g_x(\mathbf{p})$ and $g_y(\mathbf{p})$ are the first derivatives of $g(x,y)$ with respect to x and y, respectively, computed at point **p**. Here again, eq.(3) highlights gray level discontinuities with a ridge having a bell profile as well as the first absolute central moment and the top of the ridge locates the points of the discontinuity. The gradient of the GoG magnitude

$$\mathbf{n}(\mathbf{p}) = \frac{(f(\mathbf{p}) \otimes g_x(\mathbf{p}))(f(\mathbf{p}) \otimes g_{xx}(\mathbf{p})) + (f(\mathbf{p}) \otimes g_y(\mathbf{p}))(f(\mathbf{p}) \otimes g_{xy}(\mathbf{p}))}{(f(\mathbf{p}) \otimes g_x(\mathbf{p}))^2 + (f(\mathbf{p}) \otimes g_y(\mathbf{p}))^2} \mathbf{i_x} + \frac{(f(\mathbf{p}) \otimes g_x(\mathbf{p}))(f(\mathbf{p}) \otimes g_{xy}(\mathbf{p})) + (f(\mathbf{p}) \otimes g_y(\mathbf{p}))(f(\mathbf{p}) \otimes g_{yy}(\mathbf{p}))}{(f(\mathbf{p}) \otimes g_x(\mathbf{p}))^2 + (f(\mathbf{p}) \otimes g_y(\mathbf{p}))^2} \mathbf{i_y} \qquad (4)$$

indicates the path which joins point **p** to the nearest point of the nearest discontinuity.

The LoG is defined as

$$l(\mathbf{p}) = f(\mathbf{p}) \otimes (g_{xx}(\mathbf{p}) + g_{yy}(\mathbf{p})) \qquad (5)$$

where $g_{xx}(\mathbf{p})$ and $g_{yy}(\mathbf{p})$ are the second derivatives of $g(x,y)$ with respect to x and y, respectively, computed at point **p**. In this case eq.(5) highlights and locates gray level discontinuities with zero-crossings.

3. Evaluation criteria

Three kinds of errors are usually considered when the performances of two edge operators are compared: 1) the operator does not localize the edge points correctly, 2) the operator does not detect valid edge points, and 3) the operator classifies noise fluctuations as edge points. The figure of merit defined by Pratt [1] was used as a measure of performance to compare the operators since, in spite of its simplicity, it balances the three types of error introduced above and proved to be a measure which was sufficiently sensitive to the variations induced by the operators. Let us remember here the definition of the figure of merit given by Pratt:

$$R = \frac{100}{MAX(I_A, I_I)} \sum_{i=1}^{I_A} \frac{1}{1 + a d_i^2} \qquad (6)$$

where I_A and I_I represent the number of the actual and ideal edge points, respectively, a is a scaling constant and d_i is the distance between the i^{th} actual edge point and the nearest ideal edge point. A constant a equal to 1 is used in this paper. The ideal edge points, when given a test image with a gray level discontinuity affected by noise, are the points of the discontinuity and the actual edge points are the edge points located by the procedure we are testing. The ideal edge points, when given a window of a real image with a gray level discontinuity, are the points of the contour which is traced by an expert operator.

As a first step, test images with a rectilinear step discontinuity of *50 i.u.* affected by additive white Gaussian noise with variances v^2 of *100* and *400 i.u.*^{2} were used to compare the edge operators. Let r_1, r_2 and r be equal to $3\sigma_1$, $3\sigma_2$ and 3σ, respectively, where σ_1 and σ_2 are the apertures of the first absolute central moment and σ is the aperture of GoG and LoG. The results of the experimentation are illustrated in tab.1. Tab.1 shows how R changes when varying r_2 and r between *4* and *24* pixels. The table has been obtained by computing R on a region of 24X40 pixels surrounding the discontinuity over twenty samples of Gaussian noise. Therefore, for every configuration of the edge operators, each value of R is computed over *19200* points with a distance from the discontinuity which varies between *0* and *12* pixels. Tab.1 shows the results obtained on test images affected by additive white Gaussian noise with variances v^2 of *400 i.u.*^{2}. Analogous results were, however, obtained on Gaussian noise with variance v^2 of *100 i.u.*^{2}.

	$r_2 = r = 4$	8	12	16	20	24
GoG_1	22,93	47,71	88,52	100,00	100,00	100,00
GoG_2	19,42	25,61	40,06	94,52	100,00	100,00
Bar_1 $r_1=r_2$	33,97	73,13	99,26	100,00	100,00	100,00
Bar_2 $r_1=r_2$	23,41	31,34	43,60	93,27	100,00	99,81
Bar_3 $r_1=r_2/2$	82,64	97,70	99,47	100,00	100,00	100,00
Canny	24,15	66,91	99,87	100,00	100,00	100,00
LoG	16,11	27,41	48,96	91,88	94,69	96,00

Tab.1

Subsequently, images of descending thoracic aortas and peripheral arterial vessels recorded by echocardiography were used to compare the edge operators. Test windows surrounding the wall of aortas and the wall of peripheral vessels were used. The results of the experimentation are illustrated in tab.2.

	$r_2 = r = 6$	12	18	24	30
Brachial arteries					
Canny	40,26	11,61	4,30	1,69	1,37
LoG	16,82	33,86	15,12	6,54	2,63
Bar_3 ($r_1=r_2/2$)	23,83	49,62	51,40	35,98	8,33
Thoracic aortas					
Canny	22,90	17,68	4,82	1,47	0,80
LoG	14,06	28,20	25,39	11,77	5,21
Bar_3 ($r_1=r_2/2$)	18,25	39,28	31,70	5,80	2,51

Tab.2

4. Results

At first, a localization procedure which looks for local maxima of the GoG magnitude (GoG_1) and of the first absolute central moment (Bar_1) along paths perpendicular to the discontinuity was used. Let P_{G1} be the path perpendicular to the discontinuity which begins at a point *p* of a test image whose length is equal to *r*. Let P_{E1} be the analogous path perpendicular to the discontinuity which begins at *p* whose length is equal to r_2. Starting from every point *p* of the window 24X40 of the test images, the values of the GoG magnitude and the values of the first absolute central moment are computed at every point of P_{G1} and P_{E1} until the nearest local maximum is reached. If a local maximum is reached before the end of the path that point is marked as an actual edge point. Since the direction of the path is not computed, this test highlights just how much spurious local maxima generated by the GoG magnitude and by the first absolute central moment actually affect the detection and localization of the edge points.

As a second step we used a procedure which looks for local maxima along the direction of the gradient of the GoG magnitude (GoG_2) and along the direction of vector **b** (Bar_2). Let P_{G2} be the path which begins at a point *p* of a test image whose length is equal to *r* and whose direction is equal to the direction of the gradient of the GoG magnitude computed at *p*. Let P_{E2} be the analogous path which begins at *p* whose length is equal to r_2 and whose direction is equal to the direction of vector **b** computed at *p*. Starting from every point *p* of the window 24X40 of the test images, the values of the GoG magnitude and the values of the first absolute central moment are computed at every point of P_{G2} and P_{E2} until the nearest local maximum is reached. If a local maximum is reached before the end of the path that point is marked as an actual edge point. The comparison between these results and the results of the previous test highlights how the errors due to the computation of the direction contribute to the global localization error.

With a third step, the iterative procedure (Bar_3) based on the first absolute central moment was used. The comparison between the results of this test and those obtained in the second test highlights how the localization strategy contributes to the final localization error. In this case vector **b** is computed at every point *p* of the window 24X40 of the test images. Vector **b** is subsequently computed at every mass center which is obtained with the first iteration and so on. Vector **b** is computed in float precision and the relative mass center is chosen by approximating the ending point of the vector to the nearest pixel. The algorithm stops when the mass center obtained with the i^{th} iteration is the same mass center determined with the $(i-2)^{th}$ iteration, that is, when vector **b** indicates the same two pixels alternately. This is what happens at a gray level discontinuity where two contiguous pixels on opposite sides of the discontinuity are the mass centers of each other. The algorithm stops also when the mass center obtained with the i^{th} iteration is the same mass center obtained with the $(i-1)^{th}$ iteration. This happens when the magnitude of **b** is less than *0.5* pixels. If the algorithm stops before performing the N^{th}

iteration where $N=r_2$ then the last mass center is marked as an actual edge point. A mass center is marked as an actual edge point only if it is different from the starting point p of the first iteration so as to avoid marking a pixel which belongs to a homogeneous region.

Finally, with a fourth step, the localization procedures described above are compared with two standard edge detectors. Tab.1 shows the R values obtained with a Canny filter and with a LoG filter when varying the aperture σ of the Gaussian function. The two routines provided by the image processing toolbox of Matlab were used to analyze both the Canny and LoG filters on the same test images. Even though Matlab would allow us to introduce thresholds at the end of the two filtering processes the latter were set to zero since the localization procedures described above in this subsection do not use thresholds. On the other hand, the thresholding stage does not influence the comparison since, when needed, the same thresholding process (for example the Canny hysteresis algorithm) can be applied to all the procedures described in this section.

Tab.2 shows the R values obtained on windows of vascular images. These results were obtained with the iterative procedure (Bar_3) based on the first absolute central moment and with the Canny and LoG filters of Matlab.

5. Discussion and conclusions

The comparison between the results obtained with the second test and those obtained with the first test highlights how the errors due to the computation of the direction contribute to the global localization error. For small values of r_2, that is, when the computation of vector **b** introduces important errors, the fourth row of tab.1 shows smaller values than the third row. We can observe the same behavior in the second and first rows when the results obtained with the GoG operator are compared. Tab.1 also shows how the procedure based on the GoG gives rise to values of R which are, on average, slightly less than those achieved with the first absolute central moment when the standard configuration $r_1=r_2$ is used. Finally, tab.1 shows how the iterative procedure based on vector **b**, when the configuration $r_1=1/2r_2$ is used, gives rise to values of R which are much greater than the R values achieved with the absolute central moment in the second test. The iterative method based on vector **b** has two main advantages with respect to the method that looks for local maxima along the direction of vector **b**. The first advantage is that it reduces the localization errors at points which are far from any gray level discontinuity. This is because the magnitude of vector **b** is expected to be smaller than *0.5* pixel in this case and, according to the procedure described in section 4, the final mass center is the starting point itself that is not marked as an edge point. The second advantage is that spurious local maxima do not influence the result. This is an important property since the localization errors depend mainly on the presence of spurious local maxima and on the localization procedure which is incapable of avoiding these traps. Indeed, if one looks for an edge point along an erroneous direction, this does not give rise directly to a localization error. Tab.1 also shows how the iterative procedure based on vector **b**, when the configuration $r_1=1/2r_2$ is used, gives rise to values of R which are greater than the R values achieved both with the Canny filter and with the LoG filter of Matlab.

Tab.2 shows similar results. In this case also, the procedure based on the first absolute central moment provides values of R which are greater than those provided by the Canny filter and LoG. Experimental results, however, highlighted an additional problem. In echographic images the wall of the aorta, as well as the wall of the brachial artery, are reproduced as a bright bar. Therefore, the edge detectors, when applied to these images, find a pair of borders. When increasing the aperture of the Gaussian, however, GoG and LoG do not correctly localize such borders even if an ideal bar is used as a test image. Given an image with a test bar, it is easy to demonstrate that GoG and LoG, when increasing the aperture of the Gaussian, localize two parallel borders which are further apart than the borders of the test bar. Therefore, the first absolute central moment should be used to detect and localize the borders of vessels in echographic images.

References

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