

A Comparison of Continuous Wavelet Transform and Modulus Maxima Analysis of Characteristic ECG Features

I Romero Legarreta¹, PS Addison^{1,2}, NR Grubb⁴, GR Clegg³,
CE Robertson³, JN Watson²

¹Faculty of Engineering and Computing, Napier University, Edinburgh, UK

²Cardiodigital Ltd, Elvingston Science Centre, Gladsmuir, East Lothian, UK

³Department of Emergency Medicine, Royal Infirmary of Edinburgh, Edinburgh, UK

⁴Department of Cardiology, Royal Infirmary of Edinburgh, Edinburgh, UK

Abstract

The Continuous Wavelet Transform (CWT) offers a valuable tool for the analysis of signals as it provides precise location in time of high frequency components. The selection of a mother wavelet with high correlation with the signal under study provides a more accurate time-frequency analysis. Continuous Wavelet Transform Modulus Maxima (CWTMM) reduce the computational requirement by representing only the pertinent information contained within the scalogram obtained from Continuous Wavelet analysis. This new domain has an easy interpretation and offers a useful tool for the automatic characterization of the different components observed in the ECG in health and disease. The aim of this work was to compare the two time-frequency domains for ECG analysis: CWT and CWTMM, providing example applications of both methods.

1. Introduction

The surface electrocardiogram (ECG) is a crucial diagnostic investigation in many areas of modern medicine. Analysis of the ECG has become an important area of research, in particular where advanced signal processing techniques can yield useful and timely information, which is otherwise inaccessible, e.g. the identification of ventricular fibrillation during cardiac resuscitation. Traditionally, analytical tools extracting time-frequency information have been based around the Fourier Transform [1,2]. More recently, the Continuous Wavelet Transform (CWT) has been used successfully in the processing of ECG signals, and offers significant advantages – in particular the preservation of location specific features [3-5]. In this paper we explore the use of CWT and CWT Modulus Maxima (CWTMM) in the analysis of beat morphologies.

2. Theory

The wavelet transform of a continuous time signal, $x(t)$, is defined as:

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad (1)$$

where $\psi^*(t)$ is the complex conjugate of the wavelet function $\psi(t)$, a is the dilation parameter of the wavelet and b is the location parameter of the wavelet. In order to be classified as a wavelet, the function must satisfy certain mathematical criteria. These are:

1 – A wavelet must have finite energy:

$$\text{i.e. } E = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt < \infty \quad (2)$$

2 - If $\hat{\psi}(f)$ is the Fourier transform of $\psi(t)$,

$$\text{i.e. } \hat{\psi}(\omega) = \int_{-\infty}^{+\infty} \psi(t) e^{-i(\omega)t} dt \quad (3)$$

then the following condition must hold:

$$C_g = \int_0^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty \quad (4)$$

This implies that the wavelet has no zero frequency component, i.e. $\hat{\psi}(0) = 0$, or to put it another way, it must have a zero mean. Equation 4 is known as the *admissibility condition* and C_g is called the *admissibility constant*. The value of C_g depends on the chosen wavelet.

3 - For complex (or analytic) wavelets, the Fourier transform must both be real and vanish for negative frequencies.

The contribution to the signal energy at the specific a scale and b location is given by the two-dimensional

wavelet energy density function known as the scalogram:

$$E(a,b) = |T(a,b)|^2 \quad (5)$$

The total energy in the signal may be found from its wavelet transform as follows:

$$E = \frac{1}{C_g} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |T(a,b)|^2 da db \quad \left[= \int_{-\infty}^{+\infty} x(t)^2 dt \right] \quad (6)$$

In practice a fine discretisation of the continuous wavelet transform is computed where usually the b location is discretised at the sampling interval and the a scale is discretised logarithmically. The a scale discretisation is often taken as integer powers of 2, however, we use a finer resolution in our method where the a scale discretisation is in fractional powers of two. This discretisation of the continuous wavelet transform (CWT) is made distinct from the discrete wavelet transform (DWT) in the literature. In its basic form, the DWT employs a dyadic grid (integer power of two scaling in a and b) and orthonormal wavelet basis functions and exhibits zero redundancy. Our method, i.e. using a high resolution in wavelet space as described above, allows individual maxima to be followed accurately across scales, something that is often very difficult with discrete orthogonal or dyadic stationary wavelet transforms incorporating integer power of two scale discretisation. Further background information concerning continuous wavelets and their properties can be found in references [6] and [7].

Wavelet modulus maxima, defined as:

$$\frac{d |T(a,b)|^2}{db} = 0 \quad (7)$$

are used for locating and characterising singularities in the signal (Note that equation 7 also includes inflection points with zero gradient. These can be easily removed when implementing the modulus maxima method in practice). Modulus maxima-based methods are beginning to find favour in the analysis of a variety of signals including in engineering and medical signals, and the characterisation of multifractal signals [8-18].

In this study we employ the Mexican hat wavelet, which is the second derivative of a Gaussian function, defined as:

$$\psi(t) = (1-t^2)e^{-\frac{t^2}{2}} \quad (8)$$

This wavelet has been used in practice for a number of

data analysis tasks in engineering, including the morphological characterisation of engineering surfaces, the interrogation of laser-induced ultrasonic signals and the analysis of turbulent flows [7]. The Mexican Hat is used extensively in studies requiring the use of modulus maxima methods as its maxima lines (and those of other derivatives of Gaussian functions) are guaranteed continuous across scales for singularities in the signal [19].

3. Methods

A variety of ECGs with different characteristic morphologies were observed in both the CWT and CWTMM domains. Specifically, healthy ECGs with different QRS shapes [Figure 1], pathologic examples as Ventricular Tachycardia [Figure 2] and Ventricular Fibrillation were observed. No pre-processing was used.

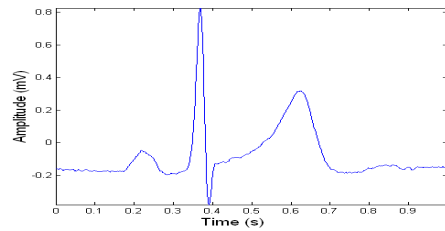


Figure 1. Fragment of 1 second length of one healthy ECG.

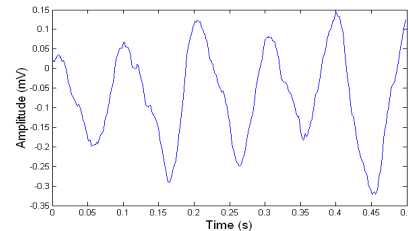


Figure 2. Fragment of 1 second length of a Ventricular Tachycardia ECG.

Simulated ECGs with added noise and small amplitude sinusoidal signals added at the end of the QRS complex were also studied [Figure 3].

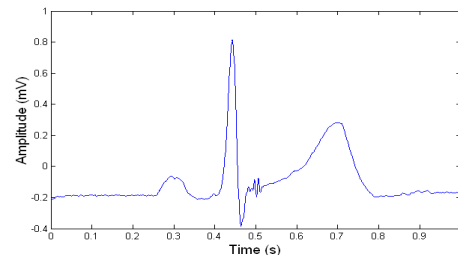


Figure 3. The ECG fragment shown in Figure 1 with an artificial signal added at the end of the QRS. No noise was added.

4. Results

The CWTMM performed well in recognising the characteristic points of the ECG including the R wave and the P and T waves whose Modulus Maxima have very recognisable characteristic shapes [Figure 4]. This domain can easily filter out the white noise within the signal.

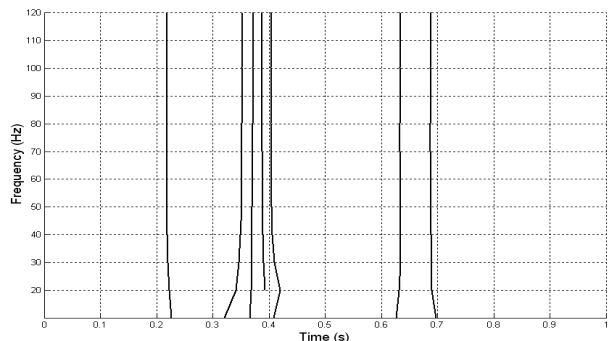


Figure 4. CWTMM of the ECG shown in Figure 1. In the figure the main characteristic waves, P, QRS and T can be clearly distinguished. The MM below 10% of the maximum value were ignored.

The CWT also performed well in this use [Figure 5]. However the information displayed in the CWTMM was clearer.

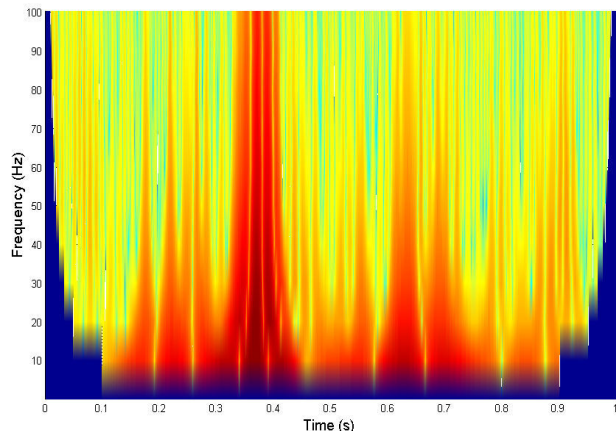


Figure 5. CWT of the ECG shown in Figure 1. In the figure the main characteristic waves, P, QRS and T can be clearly distinguished.

However the CWTMM was found not to perform as well when used to detect small oscillations added at the end of the QRS complex (i.e. simulated VLPs). These small signals can be perceived in the plot but they are difficult to differentiate from the MM associated with noise [Figure 6]. The CWT was found better for this task

[Figures 7 and 8]. In this study it was found that most characteristic waveform features within the signal can be recognized in both domains.

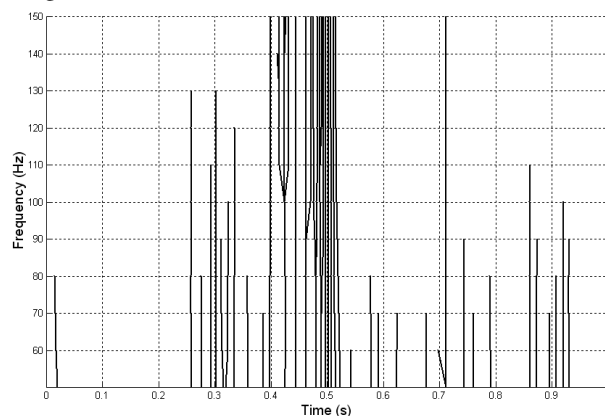


Figure 6. CWTMM of the ECG shown in Figure 3. Frequencies 50-100Hz were considered to match the artificial signals added.

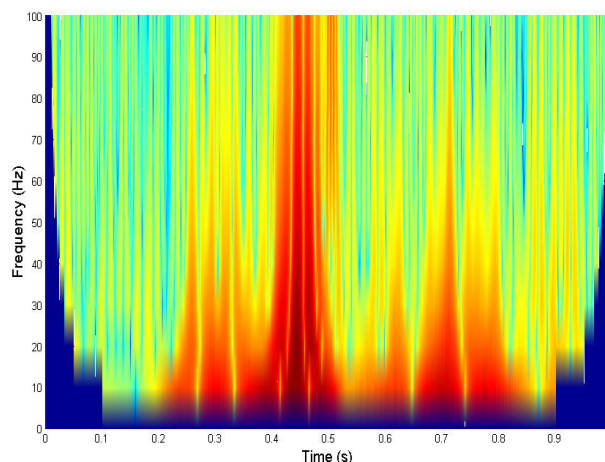


Figure 7. CWT of the ECG shown in Figure 3.

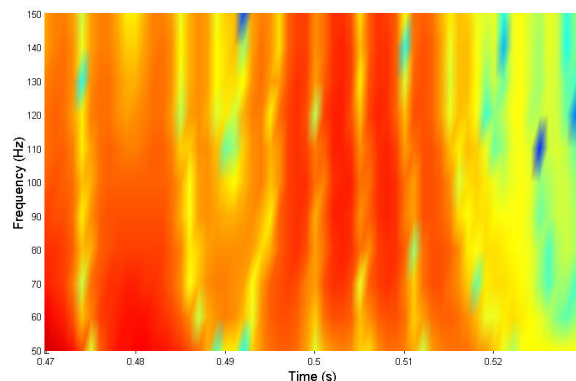


Figure 8. Zoom from Figure 7 in the temporal interval where the artificial signals were located in the time-frequency plane.

An algorithm to detect characteristic ECG points such as the R wave has been developed using the CWTMM domain and was found to exhibit good performance [20]. However VLP detection using this domain was not successful in detecting the microvoltage amplitude signals. A CWT-based method gave superior results [21] for this task.

5. Discussion and conclusions

The CWT and the CWTMM are two good tools for the analysis of ECGs and both exhibit good performance in presence of noise. The low level of computational complexity of the CWTMM makes it the easier of the two to employ in practice. However, for the detection of small waveform features within the ECG, the CWT contains more detailed information and therefore has the potential to produce enhanced results.

References

- [1] Romero Legarreta I, Serrano L, Ayesta U. ECG Frequency Domain Features Extraction: A New Characteristic for Arrhythmias Classification. 29th Annual International Conference of the IEEE Engineering in medicine and Biology Society, Istanbul, Turkey, 25-28 October, 2001.
- [2] Ayesta U, Serrano L, Romero Legarreta I. Complexity Measure Revisited: A New Algorithm for Classifying Cardiac Arrhythmias. 29th Annual International Conference of the IEEE Engineering in medicine and Biology Society, Istanbul, Turkey, 25-28 October, 2001.
- [3] Addison PS, Watson JN, Clegg G, Holzmer M, Sterz F, Robertson C. Evaluating Arrhythmias in ECG Signals using Wavelet Transforms. IEEE Engineering in Medicine and Biology, 2000, Vol.19, No.4, 383-392.
- [4] Addison PS, Watson JN, Clegg G, Steen PA, Robertson C. Finding Coordinated atrial activity during ventricular fibrillation using wavelet decomposition. IEEE Engineering in Medicine and Biology, 2002, Vol.21, 58-65.
- [5] Addison PS, Watson JN, Clegg G, Holzmer M, Sterz F, Robertson C. A Novel Wavelet Transform based Analysis Reveals Hidden Structure in Ventricular Fibrillation. Resuscitation, 2000, Vol.43(2), 121-127.
- [6] Addison PS. The Illustrated Wavelet Transform Handbook: Applications in Science, Engineering, Medicine and Finance. Institute of Physics Publishing, Bristol, 2002.
- [7] Addison PS, Watson JN, Feng T. Low-Oscillation Complex Wavelets. Journal of Sound and Vibration 2002; 254:733-762.
- [8] Kadambe S, Murray R, Boudreaux-Bartels G.F. Wavelet transform-based QRS complex detector. IEEE Transactions on Biomedical Engineering 1999 46(7), 838-848.
- [9] Watson JN, Addison PS. Spectral-Temporal Filtering of NDT Data using Wavelet Transform Modulus Maxima. Mechanics Research Communications, 2002, Vol.29, 99-106.
- [10] Watson JN, Addison PS, Grubb N, Clegg G, Robertson C, Fox KAA. Wavelet-Based Filtering for the Clinical Evaluation of Atrial Fibrillation. 29th Annual International Conference of the IEEE Engineering in medicine and Biology Society, Istanbul, Turkey, 25-28 October, 2001.
- [11] Roux S, Muzy JF, Arneodo A. Detecting vorticity filaments using wavelet analysis: about the statistical contribution of vorticity filaments to intermittency in swirling turbulent flows. The European Physical Journal, 1999,B 8(2), 301-322.
- [12] Sahambi JS, Tandon SM, Bhatt RKP. Using Wavelet Transforms for ECG Characterization: An On-Line Digital Signal Processing System. IEEE Engineering in Medicine and Biology, 1997, 16(1), 77-83.
- [13] Sahambi JS, Tandon SM, Bhatt RKP. Quantitative analysis of errors due to power-line interference and base-line drift in detection of onsets and offsets in ECG using wavelets. Medical and Biological Engineering and Computing, 1997, 35, 747-751.
- [14] Li BL, Loehle C. Wavelet analysis of multiscale permeabilities in the subsurface. Geophysical Research Letters, 1995,22(23), 3123-3126.
- [15] Bruce LM, Adhami RR. Classifying mammographic mass shapes using the wavelet transform modulus-maxima method. IEEE Transactions on Medical Imaging, 1999, 18(12), 1170-1177.
- [16] Carmona RA, Hwang WL, Frostig RD. Wavelet Analysis for Brain-Function Imaging. IEEE Transactions on Medical Imaging, 1995, 14(3), 556-564
- [17] Haase M, Lehle B. Tracing the skeleton of wavelet transform maxima lines for the characterisation of fractal distributions. Fractals and Beyond, 1998, M.M. Novak (Editor), World Scientific, Singapore,
- [18] Degaudenzi ME, Arizmedi CM. Wavelet-based fractal analysis of airborne pollen. Physical Review E, 1999, 59(6), 6569-6573.
- [19] Mallat S. "A Wavelet tour of signal processing". Editorial Academic Press, 2nd Ed, 1999.
- [20] Romero Legarreta I, Addison PS, Reed MJ et al. Continuous Wavelet Modulus Maxima Analysis of the Electrocardiogram: Beat Characterisation and Beat-To-Beat Measurement. International Journal of Wavelets, Multiresolution and Information Processing, Vol. 3, No. 1 (2005) 19-42.
- [21] Romero Legarreta I, Addison PS, Grubb NR et al. Analysis of Ventricular Late Potentials Prior to the Onset of Ventricular Tachyarrhythmias: End of QRS Point Detector. Computers in Cardiology, 2005.

Address for correspondence

Inaki Romero Legarreta
 Faculty of Engineering and Computing
 Napier University
 10, Colinton Road
 Edinburgh EH10 5DT, UK
 inaki@napier.ac.uk