Wavelet-Based Wiener Filter for Electrocardiogram Signal Denoising

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Abstract

The aim of this study is suppression of parasite electromyographic (EMG) signals (myopotentials) included in ECG signals with use of the Wiener filtering in shift-invariant wavelet domain with pilot estimation of the signal. The wavelet filtering with hybrid thresholding was used for pilot estimation. The four-levels shift-invariant dyadic discrete-time wavelet transform decomposition was used for both main blocks of pilot estimation and Wiener filtering. Sampling frequency of used signals was 500 Hz. The testing set have included signals with small waves Q, high R waves and significant variations of precipitousness in onsets and offsets of QRS complexes. These signals were additionally noised by normal distribution noise its power spectrum was adjusted according to typical form of power spectrum of EMG signals.

1. Introduction

The ECG signal is a superposition of the signal and noise. Occurrence of noise complicates a computer analysis. Linear filtering isn't suitable for wideband myopotentials suppression, because it leads to strong cut off the local extreme of QRS complexes and to disturbance the significant variation of signal precipitousness in onsets and offsets of QRS complexes.

The frequency spectrum of the ECG signal contains the components approximately from 1 to 125 Hz. Frequency spectrum of the myopotentials is sharply overlapped with spectrum of the ECG signal (approximately from 10 Hz). The intensity level of noise is low in case of rest ECG signals, it is groundless for visual analysis, however computer analysis may be complicated. More troublesome is analysis of stress ECG where the noise level is much higher then in case of rest ECG.

Discrete-time wavelet transform (DTWT) appears as a useful tool for myopotentials suppression. The filtering is based on modification of the coefficients of wavelet transform depend on estimated noise level. It can lead to minor distortion of the signal in stead of linear filtering [1]. Important is to choice a threshold strategy. Occurrence of high artefacts cause to overthreshold values of DTWT coefficients of noise is disadvantage of using a hard thresholding. It is distinct mainly around onsets and offsets of QRS complexes. On the other hand, the main disadvantage of a soft thresholding is decreasing the values of local extremes in QRS complexes and sporadic occurrence of mentioned artefacts. Smaller decreasing of local extremes and sporadic occurrence of artefacts is property of hybrid thresholding, (see Sec. 3.1).

Wavelet domain Wiener filtering with pilot estimation of the signal gives better results than wavelet filtering with using some of mentioned type of thresholding. This method do not significantly distorts the extremes in QRS complexes and it is without artefacts by realization of suitable pilot estimation. In [2] was used the wavelet domain Wiener filtering with decimation and very simplify estimation of DTWT coefficients. In [3] was realized wavelet domain Wiener filtering with pilot estimation which was composed by DTWT with decimation and hard thresholding. It have been led to frequent occurrence of artefacts in filtered signal.

The point of view in our experiments was on wavelet domain Wiener filtering with pilot estimation of the signal realized by shift-invariant dyadic DTWT. The pilot estimation have been realized as a wavelet filtering (shift-invariant dyadic DTWT) with hybrid thresholding.

2. Discrete time wavelet transform

The main block of described method is shift-invariant dyadic DTWT realized by bank of filters illustrated in the Fig. 1, where $H_{\mathrm{HP}}(z)$ is a decomposition highpass filter and $H_{\mathrm{LP}}(z)$ is a decomposition lowpass filter.

Let we assume the input data $x(n)$ as a signal $s(n)$ and additive noise $w(n)$, so $x(n) = s(n) + w(n)$. DTWT coefficients of the data $x(n)$ let we mark as a $y_m(n)$ and coefficients of the signal and noise $u_m(n)$ and $v_m(n)$ respectively, where n is an index of the coefficient of mth level of decomposition. Due to linearity of DTWT is valid $y_m(n) = u_m(n) + v_m(n)$.

By wavelet filtering is necessary to adjust the modification of the coefficients to intensity of noise components (standard deviation or variance) in mth level. When the intensity of noise was low, it would be threshold values low

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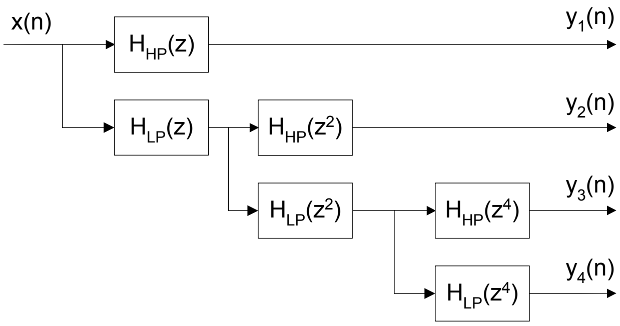

Figure 1. Three-level shift-invariant dyadic DTWT decomposition.

and the risk of the damage of signal $s(n)$ would be lower too. The variance of noise components estimation can be provided in areas between QRS complexes - length of the area is approximately 10 % of length of an R–R interval. We can expect only components of noise between QRS complexes in first 3 or 4 levels in condition of sampling frequency 500 Hz (in dependency on magnitude frequency responses of decomposition filters).

3. Wiener filtering in time-scale domain

In several publications [2, 4] can be found the analogy between modification wavelet coefficients and the Wiener filtering where the coefficients $y_m(n)$ are multiplied by suitable formfactors. It has to be sought such formfactor $g_m(n)$ as modified values ${}^{\lambda}y_m(n) = y_m(n) \cdot g_m(n) = g_m(n) \cdot [u_m(n) + v_m(n)]$ for which is valid minimum square error $e_m^2(n) = \left({}^{\lambda}y_m(n) - u_m(n)\right)^2 \rightarrow min$. Results give an equation for formfactor

$$g_m(n) = \frac{u_m^2(n)}{u_m^2(n) + v_m^2(n)} \approx \frac{u_m^2(n)}{u_m^2(n) + \sigma_{v_m}^2}, \quad (1)$$

where the noise values $v_m(n)$ are unknown, therefore their square were substituted by noise variance $\sigma_{v_m}^2$ in mth level. For $u_m^2(n) \gg \sigma_{v_m}^2$ will the $g_m(n) \approx 1$ and $|{}^{\lambda}y_m(n)| \approx |y_m(n)|$. On other hand for $u_m^2(n) \ll \sigma_{v_m}^2$ will the $g_m(n) \ll 1$ and $|{}^{\lambda}y_m(n)| < |y_m(n)|$. The coefficients $u_m(n)$ are unknown. Their estimation is possible, (see Sec. 3.2).

3.1. Hybrid thresholding

The estimation of $u_m(n)$ from $y_m(n)$ and variance of noise in form $u_m^2(n) = \max\left[k y_m^2(n) - \sigma_{v_m}^2, 0\right]$ is used in [5], where is explained the choice of constant $k = 1/3$. The result leads to formfactor

$$g_m(n) = \\ = \max\left[\frac{y_m^2(n) - 3\sigma_{v_m}^2}{y_m^2(n)}, 0\right] = \max\left[1 - 3\frac{\sigma_{v_m}^2}{y_m^2(n)}, 0\right]. \quad (2)$$

When we expressed the estimation ${}^{\lambda}y_m(n)$ with using eq. (2) like ${}^{\lambda}y_m(n) = y_m(n) \cdot g_m(n)$ we can get to notion that it is the thresholding of the coefficients with the threshold $\lambda_m = \sqrt{3}\sigma_{v_m}$, from it follow

$${}^{\lambda}y_m(n) = \left\langle \begin{array}{ll} y_m(n) - \frac{\lambda_m^2}{y_m(n)}, & \text{for } |y_m(n)| > \lambda_m \\ 0, & \text{for } |y_m(n)| \leq \lambda_m \end{array} \right. . \quad (3)$$

From eq. (3) it can be seen that it is compromise between soft and hard thresholding: it is approached to soft thresholding for values $|y_m(n)|$ approximately equal to λ_m and hard thresholding for values $|y_m(n)|$ much higher than λ_m. Therefore we named this method as hybrid thresholding.

3.2. Pilot estimation method

Other possibility of estimation the $u_m(n)$ is method of pilot estimation ${}^{p}s(n)$ of the signal $s(n)$. After DTWT decomposition we can get the coefficients ${}^{p}u_m(n)$ of signal ${}^{p}s(n)$ [4]. Principle of wavelet domain Wiener filtering with pilot estimation is shown in the Fig. 2. Realization

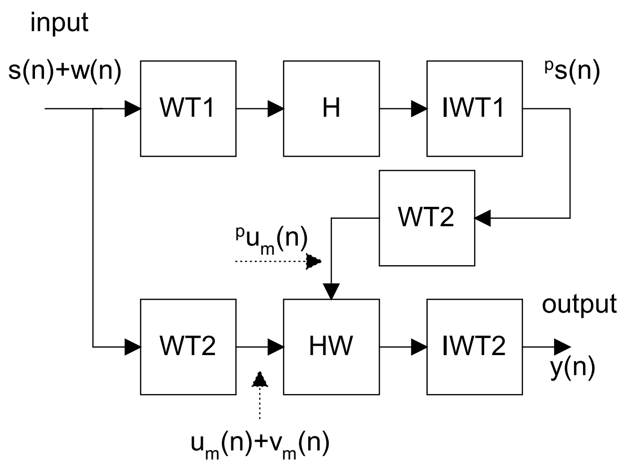

Figure 2. Principle of Wiener filtering with pilot estimation.

of pilot estimation is placed on upper branch: At first the input signal is decomposed by DTWT (WT1) into 4 levels. Than the coefficients are thresholded (block H) and reconstructed by inverse DTWT (IWT1). Output of this configuration give the pilot estimation ${}^{p}s(n)$ of the signal. Wavelet-based Wiener filtering is illustrated on the lower branch. Input signal is decomposed into 4 levels by block WT2, coefficients are modified by eq. (1) (block HW), where the $u_m(n)$ are replaced by pilot estimation ${}^{p}u_m(n)$ obtained from decomposition of pilot signal estimation ${}^{p}s(n)$ by block WT2. Output of the modification block HW is signal named ${}^{\lambda_p}y_m(n)$. Finally the inverse IWT2 is necessary to complete reconstruction of the signal $s(n)$.

For mean square error ε_y^2 between coefficients in mth level ${}^{\lambda}y_m(n)$ obtained from (1) with ideal values $u_m(n)$ and estimated coefficients ${}^{\lambda_p}y_m(n)$ can be wrote

$$\varepsilon_y^2 = \mathbb{E}\left\{\left({}^{\lambda}y_m(n) - {}^{\lambda_p}y_m(n)\right)^2\right\} = \\ \mathbb{E}\left\{\left(u_m^2(n) + \sigma_{v_m}^2\right)\cdot\left(\frac{u_m^2(n)}{u_m^2(n)+\sigma_{v_m}^2} - \frac{{}^{p}u_m^2(n)}{{}^{p}u_m^2(n)+\sigma_{v_m}^2}\right)^2\right\}, \quad (4)$$

in condition that $u_m(n)$ and $v_m(n)$ aren't correlated, so that it is valid $\mathbb{E}\left\{y_m^2(n)\right\} = \mathbb{E}\left\{u_m^2(n)\right\} + \sigma_{v_m}^2$.

Let we suppose the wavelet filtering with thresholding in branch of pilot estimation. From eq. (4) for $|{}^{p}u_m(n)| < |u_m(n)|$ results higher contributions to error ε_y^2 and for $|{}^{p}u_m(n)| > |u_m(n)|$ reversely. The sensitivity to this difference growing up in dependency of growing variation of noise. In case of $|u_m(n)| \gg \sigma_{v_m}$ than the difference between $|{}^{p}u_m(n)|$ and $|u_m(n)|$ are minimal. On other hand, in case of $|u_m(n)| \approx \sigma_{v_m}$ we can expect the retained over threshold values, however with market participation of noise. Than we can register large total error $e(n)$ between signal $s(n)$ and output signal $y(n)$. Using of the soft thresholding is more suitable in this cases. Applying of the hybrid thresholding (in block H) by eq. (3) is a compromise between hard and soft thresholding.

The hybrid thresholding with value of the threshold $\lambda_m = 3\sigma_{v_m}$ was used by pilot estimation method realization. The value of the threshold was advisedly higher against the eq. (3) in order to prevent the artefacts creation. In case of lower threshold values it exist the risk that the Wiener filter magnify the minor noise artefacts.

4. Testing set of signals and noise model

Some signals from CSE Multilead Atlas (sampling frequency $f_{\mathrm{S}} = 500\,\mathrm{Hz}$) were chosen into the testing file. Especially the signals with small Q and high R waves and with remarkable changes of signal precipitousness in QRS onsets and offsets. We have selected the signals only with minimal intensity of noise, because the signals from CSE library were discretized with quantization step $q = 5\,\mu\mathrm{V}$, a power line interference and myopotentials. This signals were preprocessed at first by Wiener filtering and than were added the autificial additive noise of known intensity. It was paid close attention to preprocessing: the result of the filtering was checked after than the signal was put into the testing set.

The additive noise is based on white noise which was frequency limited according to shape of the power spectrum of surface muscle biceps brachii EMG signal [6].

5. Discussion and conclusions

The results were assessed according to achieved signal to noise ratio $\mathrm{SNR_Y}$ of the output signal $y(n)$ by the following equation:

$$\mathrm{SNR_Y} = 10\log_{10}\frac{\sum\limits_{n=0}^{N-1} s^2(n)}{\sum\limits_{n=0}^{N-1}\left(y(n)-s(n)\right)^2} \quad [\mathrm{dB}], \quad (5)$$

where the signal $s(n)$ have had zero mean. The signal to noise ratio of the input signal $\mathrm{SNR_X}$ was computed same as eq. (5), but in denominator was only chosen variance of noise.

The different banks of filters and their combination in blocks WT1 and WT2 were tested. The orthogonal filters banks with short impulse responses (`haar`, `db2`), biorthogonal (`bior2.2`) and banks with longer impulse responses (`db5`, and `bior6.8`) were tested. Achieved results, the mean of output $\mathrm{SNR_Y}$ can be found in the Tab. 1, where the input $\mathrm{SNR_X}$ was 10 and 14 dB.

Table 1. Resultant $\mathrm{SNR_Y}$ after wavelet Wiener filtering with pilot estimation.

Banks of filters WT1 / WT2	$\mathrm{SNR_X}$ =10 dB avg. $\mathrm{SNR_Y}$ [dB]	$\mathrm{SNR_X}$ =14 dB avg. $\mathrm{SNR_Y}$ [dB]
haar / haar	21.4	24.3
bior2.2 / bior2.2	22.8	26.0
bior6.8 / bior6.8	22.8	25.9
db2 / db2	22.6	25.5
db5 / db5	21.9	24.9
haar / db2	22.1	24.8
bior2.2 / haar	21.1	25.2
bior2.2 / db2	21.2	25.5
db2 / haar	20.6	24.9
db2 / bior2.2	22.5	24.9

In cases of the same banks of filters WT1 and WT2 were achieved better results. Similar quality of filtering we can expected with combination of filters banks with short impulse responses (`haar`, `bior2.2` and `db2`). On the other side, the weakest results give the using bank of filters with long impulse responses (`bior6.8` or `db5`).

In the Fig. 3 is illustrated filtering by proposed method for input signal with $\mathrm{SNR_X} = 10\,\mathrm{dB}$ (left) and 20 dB (right). The `haar` type of filters bank in pilot estimation of the signal were used. For Wiener filtering was used the wavelet decomposition with `db2` type of filters bank. Input data are signals $s(n)$ with additive noise $w(n)$, $x(n) = s(n) + w(n)$, output of the filter marked as $y(n)$ has a total error $e(n)$ of the filtering, $e(n) = s(n) - y(n)$.

The deformations of high waves in QRS complexes in testing set of signals were depended on noise intensity and the deformation magnitude fluctuated along the whole record. The total error was commonly around tens of μV and often happened to amplification of the original values as can be seen in the Fig. 3 on the left.

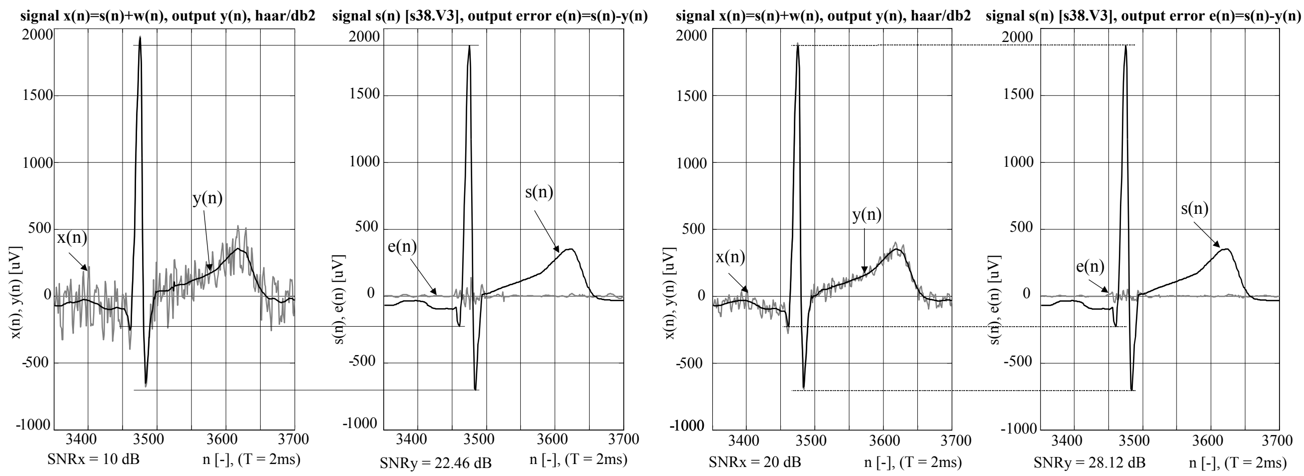

Figure 3. Filtering of the signal for SNR_X=10 dB (left) and 20 dB (right) with filters banks haar/db2.

Deformation of small waves in QRS complexes and enlarging of the QRS complexes has been proved more serious problems. With those distortions has to be supposed. Magnitude of those distortions are growing up with noise intensity (see Fig. 3). The best results were achieved with the same banks of filters used in WT1 and WT2 in agreement with global results written in the Tab. 1. In only case of using the filter banks with long impulse responses (`bior6.8/bior6.8` and `db5/db5`) were in output signal $y(n)$ visible typical oscillations placed exactly before and behind the QRS complexes.

It could not be in danger of shape distortion when this described method, with suitable chosen filters banks, is used for filtering the rest ECG signals, where is too low noise intensity. We put the preference on choice the filters bank with shorter impulse responses. Output signal could have positive effect on quality of following computer ECG analysis. The advantage of wavelet filtering in compare with linear filtering is more careful towards filtered signal. The efficiency of filtering is falling down with descending noise level. In case of stress ECG processing is the total filtration error bigger. Common measured values in stress ECG analysis is real-time trend of the ST segments monitoring. This monitoring is besides linked to quality estimation of signal level before QRS onset and behind QRS offset. QRS complexes shouldn't be after filtering dilated. Designed method we consider to useful for preprocessing the rest ECGs and stress ECGs.

Acknowledgements

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