

Characterization of Aortic Input Impedance in the Supplemental Domain

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Abstract

Although ventricular-arterial coupling has been widely studied with frequency domain modeling of arterial hemodynamics, the existence of reflected flow and pressure waves in the arterial tree still remains controversial. Aortic Input Impedance (AII) is the ratio between pulsatile aortic pressure and flow waveforms in the frequency domain, and it has been mainly measured using Discrete Fourier Series (DFS). We propose a signal processing method for accurately estimating the impulse response of AII. In an animal model, the full-band spectra of pulsatile aortic pressure and flow were calculated using homomorphic decorvolution, and then compared to DFS spectra. Given the observed band-limited nature of these signals, we formulated a supplemental domain model for AII estimation, in which calculations are made using equivalent full-band, pole-free spectra of pressure and flow signals. In three animals, impulse responses were obtained, which exhibited among-subjects reproducible patterns in the basal state. The estimation of AII in the supplemental domain allows the estimation of its impulse response, which could explain the role of reflections in the arterial tree.

1. Introduction

Basic research in cardiovascular hemodynamics is usually based on the study of the anatomy of the circulatory system, the regulatory mechanisms that control the heart and blood vessels, and the physics of blood flow. In particular, coupling of the left ventricle with the aortic arterial tree has been studied by analyzing the relationship between pulsatile pressure and flow waveforms that can be measured in the ascending aorta, which have been often considered as the sum of a forward plus a backward traveling waveform, the last being the echo of the incident wave reflected in arterial bifurcations [1]. This interpretation has stimulated research searching for the distances and intensities of those reflections [2]. However, several authors have

complained about relevant difficulties when trying to determine the left ventricular load basing only on these waveforms [2]. Furthermore, the aortic tree is now recognized as an excellent distribution network, which generates very few wave reflections [3, 4].

The Aortic Input Impedance (AII) has been widely used to measure the hydraulic load that the systemic vascular bed imposes at the ejection of the left ventricle, independently of changes in the ventricular function [5]. Let $p(t), q(t)$, be the pulsatile aortic pressure and flow waveforms in the time domain (see Figure 1). The AII is defined as the ratio between these signals in the frequency domain,

$$Z(f) = \frac{P(f)}{Q(f)} \quad (1)$$

where $P(f), Q(f)$, are the Fourier transforms of aortic pressure and flow, respectively. The most used method for estimating AII is the ratio between Discrete Fourier Series of pressure and flow waveforms [6, 7, 8]. However, this representation is limited to the low frequency band (usually not higher than 30 Hz). Moreover, this method cannot provide with an estimation of the impulse response in

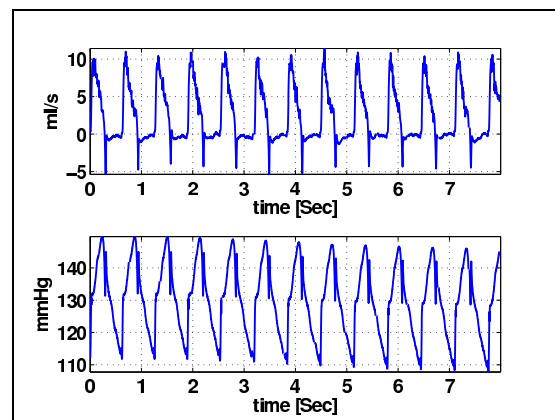


Figure 1. Examples of aortic flow (up) and pressure (down) signals in an animal model.

the time domain, which could be helpful for determining the existence of echoes, as well as for their quantification. ARMA modeling has also been proposed for estimating the AII [9], but it led to estimators with limited spectral coincidence with respect to DFS, mainly due to the spectral averaging effect of the short windows required for excluding the effect of periodicity. Therefore, a method for estimating the impulse response of AII while maintaining the coherence with DFS representation is still required.

Homomorphic system theory was introduced in the sixties [10], and it has been useful for applications in many different fields, such as speech processing, reflection seismology, radar, and medical imaging [11, 12, 13]. A constructive procedure was proposed in [14] for homomorphic deconvolution (HD) of band limited signals, that is based on adding a known (supplemental) signal in the frequency domain to obtain a full-band signal. Here, HD is used for three main purposes:

- providing with estimations of pressure and flow spectra that are coherent with their DFS representation;
- showing that the band limited nature of pressure and flow spectra is a main limitation for estimating AII, and consequently, its impulse response;
- proposing a new approach for estimating the AII in a full-band domain.

The outline of the paper is as follows. First, the mathematical foundations of AII estimation in the supplemental domain are detailed, and its particularization for a given signal is pointed out. Second, the pressure and flow spectra from an animal model signals are estimated using HD, and corresponding impulse responses are obtained for AII in the supplemental domain. Finally, conclusions and future work are presented.

2. Methods

Consider a digital signal $q[n]$ that can be expressed as the convolution of two components, i.e., $q[n] = r[n]*w[n]$. The complex cepstrum of $q[n]$ is given by

$$\hat{q}[n] = Z^{-1} \left\{ \hat{Q}(z) \right\} = Z^{-1} \left\{ \log[Q(z)] \right\} \quad (2)$$

where $Q(z)$ and $\hat{Q}(z)$ denote the Z transform of $q[n]$ and $\hat{q}[n]$, respectively, and operator Z^{-1} denotes the inverse Z transform. The complex cepstrum of $q[n]$ is the sum of the complex cepstra of $r[n]$ and $w[n]$, this is, $\hat{q}[n] = \hat{r}[n] + \hat{w}[n]$ [11]. Under appropriate conditions, the convolved components can be separated by time gating the complex cepstrum (liftering).

Usually, HD can be only used with full-band signals, because otherwise the complex logarithm is not analytical in the unit circle, and the method becomes unstable [15]. This issue has been addressed by mapping band-limited

signals into full-band ones in [14]. A full-band signal $S(z)$ (supplemental signal) is added in the frequency domain to a regularized band-limited version of the target signal, i.e., $Y(z) = Q_{rec}(z) + S(z)$, where $Q_{rec}(z) = Q(z) + \varepsilon$, with $\varepsilon \ll 1$. $S(z)$ being the sum of a full-band, minimum phase, 2^{nd} order, recursive signal $S_i(z)$, plus regularized Z transform of $q[n]$, this is, $S(z) = Q_{rec}(z) + S_i(z)$. It can be show (see [14] for details) that $\hat{q}[n]$ can be calculated from $Y(z)$.

Here, we propose to extend the previous concepts to the problem of estimating a signal $Z(z)$ that is the ratio of two band limited spectra $P(z), Q(z)$. Under these conditions, HD methods will not provide with the complex logarithm in regions of low or null spectral amplitudes. Our approach consists on mapping both band limited signals into full-band ones using supplemental signals, and to calculate $Z(z)$ in this domain (in the following, supplemental domain). First, two supplemental signals are obtained,

$$S_Q(z) = Q_{rec}(z) + S_i(z) \quad (3)$$

$$S_P(z) = P_{rec}(z) + S_i(z) \quad (4)$$

Second, full-band null-less signals are obtained as follows:

$$Y_Q(z) = Q_{rec}(z) + S_Q(z) \quad (5)$$

$$Y_P(z) = P_{rec}(z) + S_P(z) \quad (6)$$

Then, $Z(z)$ in the supplemental domain is their ratio,

$$Z(z) = \frac{Y_P(z)}{Y_Q(z)} = \frac{S_P(z) \left[\frac{P_{rec}(z)}{S_P(z)} + 1 \right]}{S_Q(z) \left[\frac{Q_{rec}(z)}{S_Q(z)} + 1 \right]} \quad (7)$$

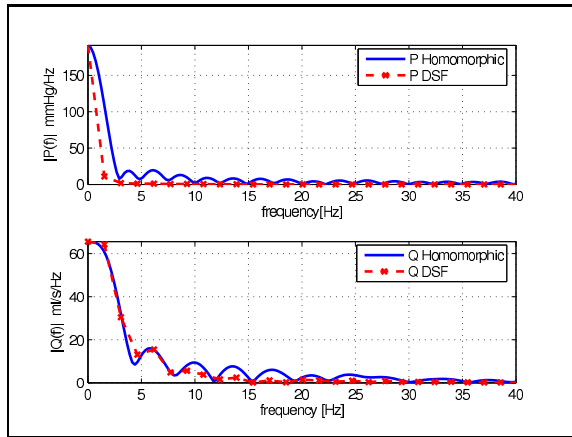
By taking the logarithm and defining

$\Phi_P(z) = \frac{P_{rec}(z)}{S_P(z)} + 1$ and $\Phi_Q(z) = \frac{Q_{rec}(z)}{S_Q(z)} + 1$, it can be shown that

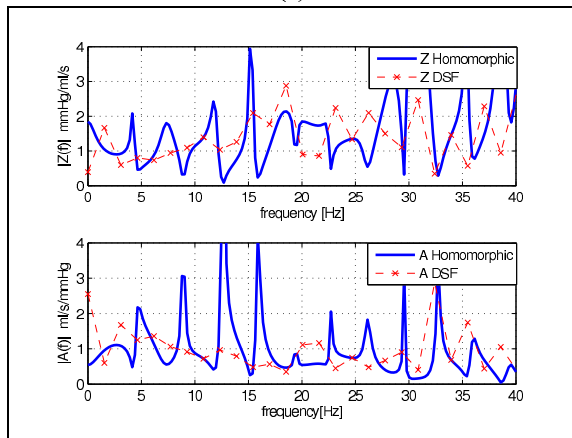
$$\begin{aligned} \hat{P}_{rec}(z) - \hat{Q}_{rec}(z) &= \log \left[\hat{\Phi}_P(z) - \hat{\Phi}_Q(z) \right] - \\ &- \log \left[\frac{P_{rec}(z)}{Q_{rec}(z)} \frac{\Gamma_P(z) Q_{rec}(z)}{S_P(z)} - \frac{Q_{rec}^2(z) \Gamma_Q(z)}{S_Q(z) P_{rec}(z)} \right] \end{aligned} \quad (8)$$

where $\Gamma_P(z)$ and $\Gamma_Q(z)$ are calculated from $\Phi_P(z)$ and $\Phi_Q(z)$, logarithm series expansion, respectively [14]. Note that the left term of the above equation corresponds to $Z(z)$ in the supplemental domain, and that the division by (close to) zero has been avoided. Moreover, the impulse response of the system can now be calculated simply by using the inverse Z transform of $Z(z)$.

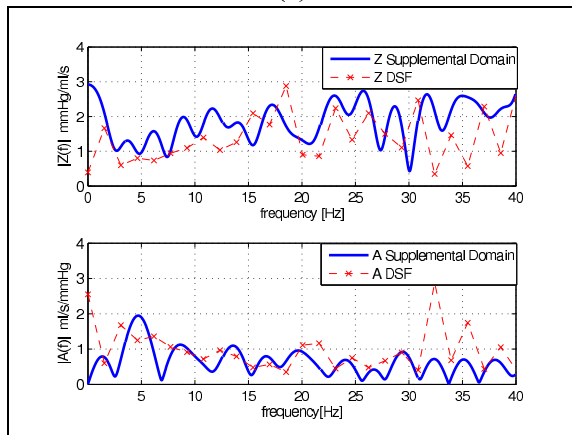
Note also that, when taking $Q(z) = 1$, the method reduces to the originally proposed one for HD of a signal.



(a)



(b)

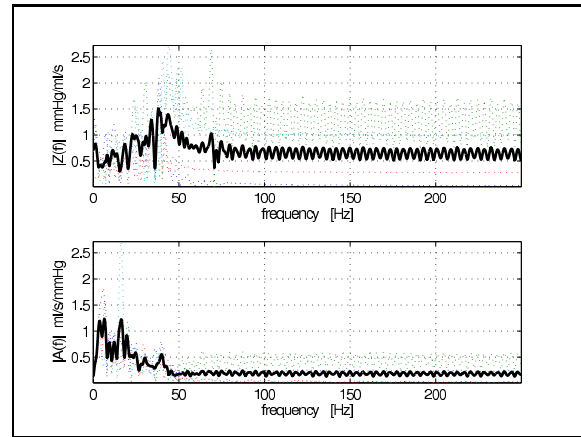


(c)

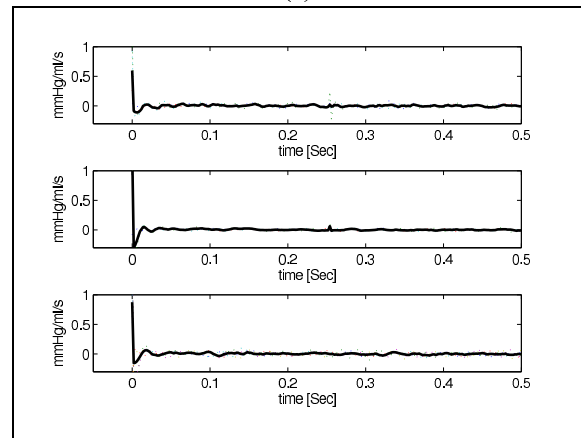
Figure 2. (a) Pressure and flow spectra. (b) AII and AIA from regularized spectra ratio. (c) The same from SD-HD.

3. Results

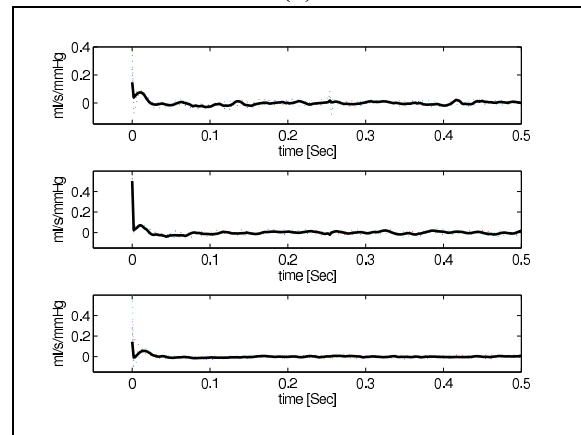
Data base and signals. Aortic pressure and flow signals (Fig. 1) were obtained from 3 healthy pigs. High-fidelity pressure recordings were obtained using a solid-state micromanometer catheter (Millar Instruments, 5F) placed



(a)



(b)



(c)

Figure 3. Averaged AII and AIA (a) and their corresponding impulse responses (b,c).

in the ascending aorta retrogradely via the carotid artery under ultrasonic guidance. Flow recordings were obtained with a flow detector using a transient-time Doppler flowmeter (Transonic Systems, Ithaca, NY) located in the ascending aorta. Flow and pressure signals were digitized

at a sampling frequency of 500 Hz, with a duration of 8 to 20 seconds.

Pressure and Flow HD. In order to obtain full-band spectral estimators of pressure and flow spectra that were coherent with their DFSs, spectral envelopes were extracted from each signal using HD [14, 16]. Free parameters of HD (length of the lifters, supplemental signal bandwidth) were adjusted for each signal according to its own heart rate. Appropriate complex cepstrum length for providing stable spectral estimators was around 400 ms in both pressure and flow.

Figure 2(a) shows that HD spectra when compared with DFS spectra exhibit a close agreement in (0,20) Hz, the usual frequency rank for DFS estimation of AII. Note that both pressure and flow spectra decrease with frequency, and more, that spectral nulls are present in the spectral envelopes. This is a non reported distortion source when estimating AII as spectra ratio, and consequently, also when estimating the impulse response. Figure 2(b) shows the regularized AII estimation from DFS and HD, together with the regularized estimation of its inverse (aortic input admittance, AIA), showing that both ratios are ill-posed problems. Similar results were obtained for all the available signals.

Impulse response estimations. The ill-posed characteristic led us to propose the calculation of the ratio of spectra in the supplemental domain, where signals are full-band and null-free. Also, the HD in the supplemental domain (HD-SD) avoids the division by zero. The HD-SD was calculated for AII and AIA in each pig. Figure 2(c) shows that, for the previous example, the effects of nulls and of low spectral amplitudes have been compensated in both impedance and admittance.

Under these conditions, it is possible to obtain the impulse responses of AII and AIA. For each 8 sec recording, a 3 sec overlapping window of 4 sec was applied, and HD-SD were obtained for each segment. Figure 3(a) shows an example of segmental and averaged spectral envelopes. Figure 3(b,c) shows the averaged impulse response for each pig. AII corresponded to a high-pass system, whereas AIA corresponded to a low-pass system. Although no reflector spikes appear in any of them, this fact should still be interpreted cautiously, as sensitivity to phase unwrapping was observed. Nevertheless, the reproducibility among subjects of the impulse response waveforms was excellent.

4. Discussion and conclusions

A new approach has been proposed for estimating the impulse response of AII, by using HD in a full-band supplemental domain. Further research has to be dedicated to enhance the robustness of the method (specially in the phase-recovery stage), as well as to relate the impulse re-

sponse with different pathologies.

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