Time –Frequency Wavelet Transform Coherence of Cardio-Respiratory Signals during Exercise

K Keissar, LR Davrath, S Akselrod

Tel Aviv University, Tel Aviv, Israel

Abstract

Wavelet Transform Coherence (WTC) is a method originally introduced to discover transient linear correlations in random processes. We adjusted and modified the WTC to establish it as a tool for the evaluation of the Autonomic Nervous System (ANS) activity by applying a bivariate analysis of the cardio-respiratory signals. The WTC provides insight into the transient linear order of the system through the computation of time-frequency maps. Quantitative methods for specific time or frequency bands are also introduced. The WTC was implemented on data from an experimental protocol which included Graded Exercise Stress Testing (GXT) with 8 normal subjects. The WTC time-frequency map exhibited a novel presentation of the dynamical changes in RSA phenomena. Furthermore, an integration over the respiratory frequency band revealed a significant (p<0.001) drop of HR–Respiration coherence around 75% of the maximum effort.

1. Introduction

Spectral analysis of heart rate (HR), respiration and blood pressure (BP) signals, is a well established tool for the noninvasive investigation of cardiovascular and cardiorespiratory control mechanisms (1). Changes in frequencies above 0.04 Hz provide evidence for the active existence of either sympathetic or parasympathetic control mechanisms (2). The relevant space is divided into the Low Frequency (LF) band (0.04 - 0.18 Hz) and to the High Frequency (HF) band (0.18 – 0.4 Hz) which includes the typical respiratory frequency for adults.

The interrelation and influence between HR, BP and Respiration signals also provide information about the functionality, about the ANS peripheral reflex mechanisms, and about the origin and characteristics of the driving sources. One strong inter modulation is the modulation of the heart rate in relation to the respiration cycle, known as the respiratory sinus arrhythmia (RSA) (3). Even though the relations between the ANS and the responses of the cardiovascular and respiratory system are yet to be fully understood, it is a fair assumption to say that most of the regulatory mechanisms have a strong linear order. Therefore, many methods of investigation of the regulatory system examine linear relations or their different properties (linear feedback, correlation and phase) (4). The Coherence function is a straightforward method to quantify the existence and strength of a linear coupling between the signals in the frequency domain. As such it is applied as a common tool in the analysis of the ANS functionality via the examination of the cardiovascular and respiratory behavior (4).

A major limitation of the coherence function is the steady state assumption. This stipulation limits the analysis to periods or conditions with evident steady state exterior conditions. Therefore it requires some level of disregarding the dynamic, complex and noisy characteristics of the living physiology. The Wavelet Coherence combines the high efficiency time frequency decomposition of the wavelet transform with the marker of linear coupling strength, the Coherence function. In this study we modified and applied the Wavelet Coherence to a case of cardio-respiratory behavior in order to establish the Wavelet coherence as a fundamental method in Physiology research, particularly in ANS.

2. Methods

2.1. The Wavelet Transform (WT)

This work applies a continuous wavelet function based on the Morlet wavelet function, consisting of a plane wave modulated by a Gaussian.

$$\Psi_0(\eta) = \pi^{-\frac{1}{4}} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2} \tag{1}$$

$$\Psi_S(t) = s^{-\frac{1}{2}} \Psi_0(t/s) \tag{2}$$

$$\eta = t/s \tag{3}$$

where η is the nondimensional time of the WT and ω_0 is its nondimensional frequency respectively. t is the time and s is the Scale. The nondimensional frequency ω_0 is in fact the Morlet wavelet coefficient. For physiological signals, good results are achieved with ω_0 in the 6-20 range.

The continuous wavelet transform (CWT) is defined by the convolution of a scaled parent wavelet function (2) with the analyzed function *g* (t).

$$W(s,\tau) = \int g(t)\Psi_s(t-\tau)dt \qquad (4)$$

The more relevant form of the transform is the continuous wavelet transform of a discrete sequence x_n.

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \Psi^* \left[\frac{(n'-n)\delta t}{s} \right] \qquad (5)$$

$$\Psi \left[\frac{(n'-n)\delta t}{s} \right] = \left(\frac{\delta t}{s} \right)^{\frac{1}{2}} \Psi_0 \left[\frac{(n'-n)\delta t}{s} \right] \qquad (6)$$

where Ψ^* is the complex conjugate of the normalized wavelet function, n is the time series index, δt is the sampling time and s is the scale. The wavelet power spectrum estimator of x_n is defined in the naïve way

$$W_n^{xx}(s) = W_n^x W_n^{x*} \qquad (7)$$

where W_n^* is the complex conjugate of the wavelet coefficient calculated in (5)

The relationship for Morlet Wavelet between the scale and the frequency is established through the Fourier wavelength λ (5)

$$\lambda = \frac{1}{f} = \frac{4\pi s}{\omega_0 + \sqrt{2+\omega_0^2}} \qquad (8)$$

2.2. Wavelet Coherence (WTC)

The Coherence function of two time series signals is defined as their cross correlation also known as the cross – spectral density normalized by the autospectral density of the two original signals.

$$\gamma_{xy}^{\ 2} := \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)} \qquad (9)$$

Where x and y are the two time series signals. Corresponding with the wavelet power spectrum, the cross wavelet spectrum of two signals x,y is

$$W_n^{xy} = W_n^x(s)W_n^{y*}(s) \qquad (10)$$

A common (6;7) squared wavelet coherence estimator is defined as the absolute value squared of the smoothed cross wavelet spectrum, normalized by the smoothed wavelet power spectrum of the two signals

$$\hat{C}_n^2(s) = \frac{\left|\left\langle W_n^{xy}(s)\cdot s^{-1}\right\rangle\right|^2}{\left\langle \left|W_n^{xx}(s)\right|\cdot s^{-1}\right\rangle \left\langle \left|W_n^{yy}(s)\right|\cdot s^{-1}\right\rangle} \qquad (11)$$

The smoothing operator indicated by the < > brackets is achieved by a convolution in time and scale. In order to obtain the optimal smoothing with minimal reduced localization, the convolution is computed in relation to the decorrelation length. The time convolution is done with a Gaussian $e^{-\frac{t^2}{2s^2}}$ which is the absolute value of the wavelet function in each scale (first term in Eq. 12).

The scale convolution is performed by a rectangular window with a length of $\delta j_0 \cdot s$ (second term in Eq. 12), where $\delta j_0 = 0.6$ is the empirical scale decorrelation length for Morlet wavelet (5).

$$\left\langle W \right\rangle = [(c_1 w_n(s) * e^{\frac{-n^2}{2s^2}})_n * c_2 \prod (\delta j_0 s)]_s \qquad (12)$$

where c_1 and c_2 are normalization factors and $\prod$ is the rectangular function.

Three useful presentation of the WTC are the time - frequency coherence power map, the time averaged coherence (13) and the scale (or frequency) averaged coherence(14).

$$C^2(f) = \frac{1}{n_2 - n_1} \sum_{n_1}^{n_2} C^2_{sig}(n,f) \qquad (13)$$

$$C^2(n) = \frac{1}{\log_2(f_1/f_2)} \sum_{f_1}^{f_2} C^2_{sig}(n,f) \qquad (14)$$

The averaging coefficient in Eq. (14) is due to the logarithmic resolution of the frequency. C_{sig} is the coherence value wherever it is considered significant (above threshold).

2.3. WTC threshold

In order to determine the existence of coherence, we choose a null hypothesis that the signals are not coherent at the considered frequency. Using a magnitude threshold, all the regions having values which are significantly far from the values typical for the coherence of uncoupled signals, will be considered as regions in which a linear coupling exists between the signals. The numerical solution is to use surrogate data (random white noise signals) for the estimation of the null hypothesis. The distribution was calculated for 100 0000 values (in segments of 5000) for each frequency and the critical level was set at 95%.

2.4. Patients and protocols

The manifestation of the Wavelet Coherence was investigated using a group of 8 healthy male subjects, age 41±8. The group served as a control group in the investigation of the contribution of the ANS to heart rate recovery (HRR) after an exercise test (8). The study was approved by the Helsinki committee of Maccabi Health Insurance. All subjects signed a written informed consent form before participation. Subjects performed a Graded Exercise Stress Test (GXT) using the Bruce protocol which was terminated when they reached 90% of age-predicted maximal HR. Data was recorded continuously during a 10 min seated rest period, the entire GXT and during the 10 min recovery period. 40 seconds after the termination of exercise, patients were seated in a chair placed on the treadmill and remained seated during recovery.

The signals extracted during this protocol were: ECG 6 leads and RIB/ABDOMINAL (ABD) respiratory signals.

Data Acquisition and pre-processing: ECG and Respiration were monitored non-invasively using a BIOPAC A/D system: ECG - using the BIOPAC preamplifier and Respiration - using the Respitrace pneumoplethysmograph impedance belts. HR signal was obtained by detecting R waves timing, by correcting eventual arrhythmias and creating a continuous 10Hz signal by interpolation. All signals were low pass filtered and decimated to 10 Hz before any analysis.

2.5. Data analysis and statistics

All the data in the time areas of interest was analyzed in the time frequency domain applying Morlet Wavelets and the time frequency Coherence. The WTC procedure was applied to HR-RIB signal couples. The analysis focused on the HF bands but is stretched between 0.02 – 2 Hz. all analyses were performed with $\omega_0 = 20$.

Significance calculations between different time regions were obtained using the student t-test of two samples with unequal variance. Results were considered significant for $p<0.05$.

3. Results

The HF band centered around the respiratory frequency is typically defined around 0.18 – 0.4 Hz, but since respiration rate increases as exercise intensity increases, the HF band follows this increase, as clear from Figure 1. After reviewing the coherence maps of all the subjects, we decided to extend the HF band to 1 Hz to include the entire respiratory frequency band.

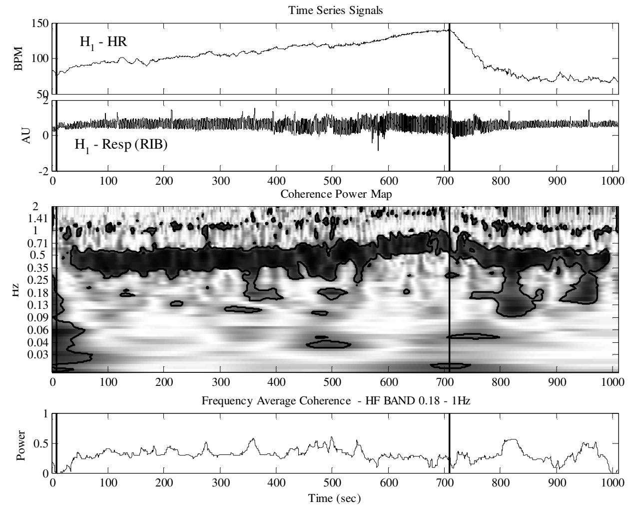

Figure 1 : H_1 subject's example of GXT with continuous Coherence band. The two top panels are the HR and Respiration signals. The middle panel is the Coherence map of HR – RESP (RIB) and the lower panel is the time average frequency dependent coherence of the extended HF Band (0.18 – 1 Hz). The two vertical lines represent the beginning (near 0 sec) and end (near 600 sec) of the GXT.

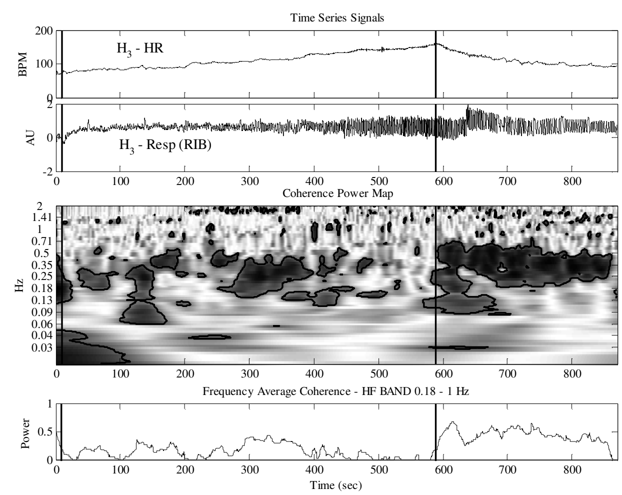

Figure 2: H_3 subject's example of GXT with a fragmented Coherence during the GXT and a continuous one after.

Individual observation of the Coherence maps showed a clear existence of coherence in the HF band which followed the increase in respiration rate but did not change much in frequency width (Figure 1). Five subjects showed almost continuous Coherence and three subjects had discontinuous coherence with no coherence during about 50% of the analyzed time (Figure 2). After the end of the exercise the subjects were seated for rest and again five individuals (not the same five as before) showed continuous coherence which of course descended to the normal HF band (Figure 1& Figure 2) while three individuals had a more fragmented coherence band during rest.

The general coherence trend during the GXT and the following recovery was analyzed by observing the average HF band WTC for the eight subjects. Since each subject finished the exercise at a different timing, we normalized the timing of the GXT of each subject by their recorded onset and end timing. Since the recovery period is the same for all subjects, it is presented in seconds (Figure 3). The average HF band coherence starts low at the onset of the exercise, probably due to loss of coherence in the transition from sitting to running on the treadmill. During the exercise, coherence at HF is building up and somewhere between 75% of effort to max the coherence starts to weaken significantly (p<0.001). After the subject begins to recover, the coherence starts to build up again for about 1 – 2 minutes.

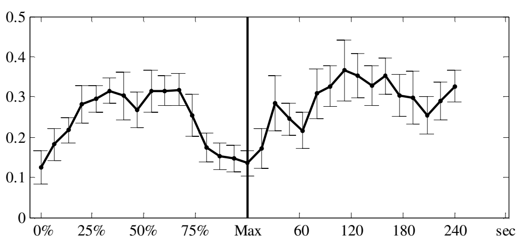

Figure 3: group average of the WTC in the HF band with std. HR – RESP for the GXT fallowing a 240 sec of seated rest. Note that the X axis is not linear. The left area is normalized for all subjects by their GXT individual timing and the right area is in seconds. The vertical line represents the end of the test and a shift to a seated rest.

4. Discussion and conclusions

We have developed a new approach to compute coherence during non steady state conditions. We have termed it time – frequency coherence: WTC. The results obtained with this new tool were clear and easy to interpret, providing a new insight into the dynamics of order and disorder in the cardio-respiratory system.

The physiological situation examined, follows the dynamic conditions of the GXT. Coherence exhibits itself throughout the active exercise which includes a dynamic increase in the frequency of the respiratory HF band (Figure 1). This coherence band gives a dynamic presentation of RSA of the tested subject during the GXT.

The average HR – RESP coherence of the entire group exhibits a significant (p<0.001) reduction in coherence long before the maximum effort (Figure 3). We believe that this reduction, which ends when the subject starts recovery, is due to changes in ventilation related to reaching and surpassing the anaerobic threshold. This behavior could be an alternative marker for easy identification of the anaerobic threshold (9).

In conclusion, the WTC is an intuitive, straightforward complementary tool in the multivariate spectral analysis of the ANS regulatory mechanisms in health and disease. WTC can be used as a direct tool for dynamic analysis or for detection of irregularities in data when a priori assumptions of linear coupling are made

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Address for correspondence

Professor Solange Akselrod
Department of Medical Physics, Tel Aviv University
Tel Aviv, Israel 69978
solange@post.tau.ac.il