

# Generalized Distribution and Q Statistics Evidences in Heart Rate Variability

LO Murta Jr<sup>1</sup>, KC Nakzato<sup>2</sup>, L Gallo Jr<sup>2</sup>

<sup>1</sup>Department of Physics and Mathematics - FFCLRP University of São Paulo, Ribeirão Preto - SP Brasil

<sup>2</sup>Medical School of Ribeirão Preto University of São Paulo, Ribeirão Preto - SP Brasil

## Abstract

*Tsallis [Physica A, 340 (2004) 1] identified a set of numbers “q-triplet” = (q-stat, q-sen, q-rel) suitable for a complex system with scale-invariance feature. When  $q = 1$ , the generalized q-Gaussian assumes classical Gaussian distribution form. Deviation of the  $q$  from unit is a measure of the departure from thermodynamics equilibrium.*

*This work presents q-stat, one of the q-triplet numbers, for heart rate variability (HRV) system. The q-stat states for the probability distribution of a system in phase space. A collection of normal subjects HRV data was fitted to generalized q-Gaussian distribution to find out which  $q$  value correspond to the dynamical system. The heart rate signals were obtained from the PhysioBank Normal Sinus Rhythm RR Interval Database. The results have shown a tending number for generalized parametric value for  $q$  systematically different from unit when distributions were fitted by q-Gaussian. Findings from the results suggest that generalized q-statistics is more suitable for HRV system investigations.*

## 1. Introduction

One of the most important contributions of information theory introduced by C. E. Shannon in 1948 [1] was the concept of entropy as information quantity. This concept has been successfully used to quantify system information since then, in a large range of applications.

Information can be quantified as follows. If  $X$  is the set of all messages  $x$  that  $X$  could be, and  $p(x) = Pr(X = x)$  is the probability of a message  $x$ , then the entropy of  $X$  is:

$$H(X) = - \sum_{x \in X} p(x) \log p(x)$$

The entropy formula was conceived by Shannon intuitively rather than from basic principles. We can conjecture, for instance, that a message with complete

certainty of occurrence, i.e. probability equals to unit, the net entropy is zero. On the other hand, if a message is rare, with very small probability of occurrence, the net entropy is large instead. The mathematical function that has this behavior is logarithmic function.

The choice of logarithmic base in the entropy formula determines the unit of information entropy that is used. The most common unit of information is the bit, based on the binary logarithm. An interesting and useful property of entropy is the fact that, for a closed dynamic system, the entropy always grows to a maximum. This can be used in an optimization algorithm, for example.

This formalism has been shown to be restricted to the domain of validity of the Boltzmann–Gibbs–Shannon (BGS) statistics. These statistics seem to describe nature when the effective microscopic interactions and the microscopic memory are short ranged. Generally, systems that obey BGS statistics are called extensive systems. If we consider that a physical system can be decomposed into two statistical independent subsystems  $A$  and  $B$ , the probability of the composite system is  $p^{A+B} = p^A \cdot p^B$ , it has been verified that the Shannon entropy has the additivity property:

$$S(A+B) = S(A) + S(B)$$

However, for a certain class of physical systems, which entail long-range interactions, long time memory and fractal-type structures, some kind of extension appears to become necessary. Inspired by multifractals concepts, Tsallis has proposed a generalization of the BGS statistics. The Tsallis statistics is currently considered useful in describing the thermostatical properties of nonadditive systems, and it is based on a generalized entropic form,

$$S_q = \frac{1 - \sum_{i=1}^k (p_i)^q}{1 - q},$$

where  $k$  is the total number of possibilities of the system and the real number  $q$  is an entropic index that characterizes the degree of nonadditivity. This expression

meets the BGS entropy in the limit  $q \rightarrow 1$ . The Tsallis entropy is nonadditive in such a way that for a statistical independent system, the entropy of the system is defined by the following pseudo additivity entropic rule

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q) \cdot S_q(A) \cdot S_q(B)$$

Considering  $S_q \geq 0$  in this pseudo-additive formalism, three entropic classifications are defined as follows

- Subadditive entropy ( $q < 1$ )  $S_q(A+B) < S_q(A) + S_q(B)$
- Additive entropy ( $q = 1$ )  $S_q(A+B) = S_q(A) + S_q(B)$
- Superadditive entropy ( $q > 1$ )  $S_q(A+B) > S_q(A) + S_q(B)$

Limit theorems, in particular, the central limit theorems (CLT), surely are among the most important theorems in probability theory and statistics. They play an essential role in various applied sciences as well, including statistical mechanics. Historically A. de Moivre, P.S. de Laplace, S.D. Poisson and C.F. Gauss have first shown that Gaussian is the attractor of independent systems with a finite second variance. Chebyshev, Markov, Liapounov, Feller, Lindeberg, Levy have contributed essentially to the development of the central limit theorem.

Within the nonextensive paradigm the CLT assumes another form with a new distribution q-Gaussian [2].

## 2. Methods

The heart rate variability data were extracted from Physionet data bank, normal sinus rhythm. The data probability density function was best fitted to a q-Gaussian with a varying q parameter. The resulting q is the q that results to a best linear regression technique, according to the equation:

$$e_q^x = [1 + (1 - q)x]^{1/(1-q)}$$

Note that this equation allows different q values. Therefore one can vary the q parameter and choose the best fit.

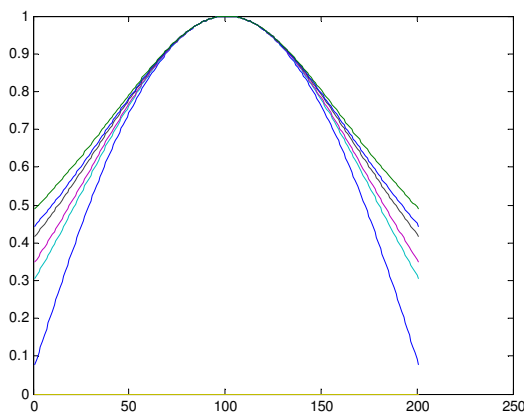


Figure 1: q-Gaussians with q distribution parameter varying from 0.1 to 3.1.

## 3. Results

The results are obtained from the probability density from each individual data, consisting on R-R interval time series from 48 hours holter exams. All the analysis were made on normal sinus rate dataset in Physionet data bank.

Given the dataset studied, the value found for q entropic parameter was  $1.58 \pm 0.76$  (mean value  $\pm$  standard deviation) showing a clear tendency apart from 1. In figure 1 shows several q-Gaussians with entropic parameter ranging from 0.1 to 3.1.

## 4. Discussion and conclusions

The results obtained in this work suggest that the nonextensive statistical paradigm may supply a good methodology for heart rate variability analysis. The achieved q parametric value for R-R intervals distribution is consistent with the nonextensive paradigm indicating the well known strong correlation between the past and future events in heart rate variability and the fact that the heart rate variability system generates a multifractal featured with scale invariance signal.

The result suggests that nonextensive paradigm is adequate to interpret and analyses the heart rate dynamics. This finding may encourages further studies and analysis methods.

## Acknowledgements

This work has been supported by the São Paulo State Foundation for Research Support (FAPESP) under project number 2006/00723-9.

## References

- [1] Shannon CE. A Mathematical Theory of Communication. Bell System Technical Journal 1948; 27: 379-423.
- [2] Umarov S, Tsallis C, Steinberg S. A Generalization of the Central Limit Theorem Consistent with Nonextensive Statistical Mechanics. 2006: arXiv:cond-mat/0603593v3.

Address for correspondence

Name Luiz Otavio Murta Junior  
 Full postal address Department of Physics and Mathematics,  
 University of São Paulo, Av. Bandeirantes 3900, 14040-901,  
 Ribeirão Preto (SP), Brazil.  
 E-mail address murta@ffclrp.usp.br