

A Mesh-less Approach for Fast Estimation of Electrical Activation Time in the Ventricular Wall

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Abstract

The dynamics of electrophysiological wave propagation in ventricular tissue are the result of a large number of interrelated processes. However there are applications in which a rough estimation of tissue activation time at a set of given points is sufficient. This paper presents a mesh-less method for fast calculation of the activation time at any arbitrary point of the ventricular domain. It is based on estimating the path and the elapse time that an electrophysiological signal would need to travel over two points on a given 3D geometry. Due to the inhomogeneities of the medium and its layer based structure, the ventricular wall is modeled as a multi-layered domain. The travel time of a wave is estimated by finding the trajectory that it would follow in a multilayered region. The performance and accuracy of the method is checked against the Eikonal model in a two layer axisymmetric left ventricle. The comparison shows some benefits from the current approach. This method is useful for the estimation of the electrical activation sequence in a hexahedral mesh during mechanical simulation of the heart, since it removes the need for another refined mesh for the electrophysiological part.

1. Introduction

Studying the pathophysiology of the heart using computer models has drawn much attention to researchers over the last years. Computer electrophysiology models are developed to improve the understanding and procedures for cardiac therapies such as cardiac resynchronization therapy (CRT). In particular, computer models are being used to plan CRT procedures, analyzing the best CRT pacing strategies [1], and lead placement [2]. There is an increasing number of models that are able to mimic the complex process of cardiac electrical activation *in-silico*, to study pathologies or improve intervention guidance and planning [3]. These models are applied to investigate electrophysiological cardiac phenomena such as cardiac arrhythmia or

fibrillation, which requires to explicitly model membrane ionic currents [4]. Nonetheless, the level of the details required to model the dynamics of action potential propagation in ventricular tissue depends highly on the ultimate target application [5]. This is the case of simple electrical wave propagation models that are designed for fast intra-operative decision making [6]. These models are based on the assumption that the speed of propagation varies more slowly and over much larger spatial scales than the transmembrane potential [7].

Simulation studies have shown the importance of modeling the fast conduction system, or at least the Purkinje network (PK), in order to obtain physiological depolarization sequences [4]. This was one of the most important drawbacks in previous studies that based their electrical modeling in fast models such as the Eikonal.

The purpose of this manuscript is to present an approach for fast modeling of the electrical wave propagation on ventricular tissue. The proposed model has the advantage of being mesh-less, therefore does not require fine computational meshes and can share the mesh used for computational mechanics. The effect of PK and the myocardial fiber orientation is also modeled. Here, we present the model description and the implementation procedure.

2. Methods

The electrical activation starts at a given initial stimulation point and propagates with a velocity that is a function of tissue type. Therefore, we assume that the onset of the action potential is distributed in the tissue as a function of time, distance from the source and tissue conduction velocity coefficients. The aim is to estimate the path that this electrical wave would follow through several layers of tissue. The formulation is derived for a three-dimensional cubic tissue block from the ventricular wall (Figure 1). The goal is to calculate the activation time for any arbitrary point in the domain using a layer map that includes the local conduction velocity. The geometry of the model is similar to the hexahedral elements that are commonly used

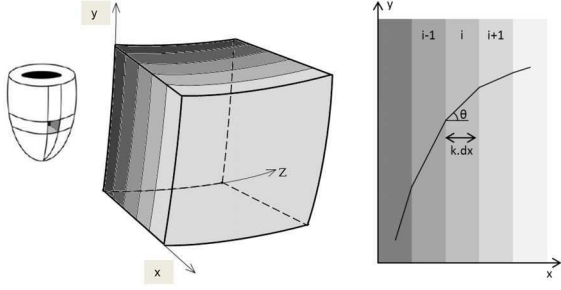


Figure 1. Wave propagation in a spectrum of transposed ventricular layers with different conductivities.

for mechanical meshes [2]. With this formulation the activation time map can be calculated on the coarse mesh generated for mechanics. In order to combine the fast propagation effects of Purkinje fibers and direction of myofibers, we use the superposition principle to sum their effects. We assume an isotropic conductivity and constant conduction velocity v_p for the purkinje fibers [8]. Measurements show that for the case of a dense PK the electrophysiological impulse propagation could be considered as homogenous [9]. We assume myocardial tissue as transversely isotropic material where the plane of isotropy is along the layers. Anisotropic conductivity is changed to isotropic conductivity by modifying the thickness of the layers from dx to an appropriate thickness of $dx' = k dx$ where k is defined as,

$$k := \frac{v_p + v_{fi}}{v_p + v_{ft}}, \quad (1)$$

where v_{ft} and v_{fi} are the conduction velocity in transmural and in-plane directions respectively. In the isotropic domain, we can expect a linear trajectory of the waves as shown in Figure 1. Since the velocity of the signal is different in each layer, the path of the wave will change according to the fermat's least time principle. It could be shown mathematically that, according to this principle, the time functional minimization will lead to Snell's law below;

$$\frac{\sin \Theta_i}{\sin \Theta_{i+1}} = \frac{v_i}{v_{i+1}}, \quad (2)$$

here v_i and v_{i+1} are the wave conduction velocities in the layers i and $i + 1$ and Θ_i and Θ_{i+1} are the angles between the normal (to the interface) plane and the incident waves respectively. v in the layers is defined as below:

$$v := v_p + v_{fi}, \quad (3)$$

Ionic waves follow Eq. (2) at the interface of two media with different diffusion coefficients [10]. For a multi-layer domain, we have,

$$\frac{\sin \Theta_{i-1}}{v_{i-1}} = \frac{\sin \Theta_i}{v_i} = \frac{\sin \Theta_{i+1}}{v_{i+1}} = c, \quad (4)$$

or,

$$\sin \Theta_i = c v_i, \quad i = 1, \dots, N \quad (5)$$

where N is the number of layers and c is a constant that depends on the path of the trace and diffusivity in the layers. c is given by the state relation $dy = \tan \Theta k dx$ where in the case of very thin layers, $dx \rightarrow 0$ and using Eq. (5), it can be written as,

$$\int_0^h \frac{c k v}{\sqrt{1 - c^2 v^2}} dx = b. \quad (6)$$

Eq. (6) is valid for a wave that starts at the origin and propagates to an arbitrary point (h, b) . h is the depth of the target point in the myocardial wall measured from the endocardium, and b is the distance from the stimulation point to the projection of the target point on the endocardium surface. Having c , the required time to cross layer i is given by $dt = \frac{k dx}{v \cos \Theta}$, then in a continuum region the required time t to propagate to the point (h, b) would be,

$$\int_0^h \frac{k}{v \sqrt{1 - c^2 v^2}} dx = t, \quad (7)$$

In the case of domains that contain both continuous and discrete conductivity layers, Eqs. (6) and (7) could be written as,

$$\sum_{i=0}^n \frac{c k_i v_i dx_i}{\sqrt{1 - c^2 v_i^2}} + \int_0^h \frac{c k v}{\sqrt{1 - c^2 v^2}} dx = b, \quad (8)$$

and

$$\sum_{i=0}^n \frac{k_i dx_i}{v_i \sqrt{1 - c^2 v_i^2}} + \int_0^h \frac{k}{v \sqrt{1 - c^2 v^2}} dx = t, \quad (9)$$

where n is the number of layers with constant conductivity coefficient. Having the geometrical parameters (h, b) and the function v , the activation time could be calculated from Eqs. (8) and (9). The sequence of calculations for an arbitrary point p is described in algorithm I.

Algorithm I

1. Estimate the velocity map v_p and v_f
2. Calculate the values of v and k
3. Find geometric parameters b and h
4. Calculate c from equation (8)
5. Calculate t from equation (9)

In order to model the conduction velocity map, here we assumed that two factors influence the electrophysiological propagation in the heart, i) the myofibers arrangement, and ii) the Purkinje network. Myofiber arrangement influences the direction of propagation, so they are modeled as inhomogeneous conductive parts. Purkinje network branches are modeled as an isotropic region located at the subendocardial region of the heart [8].

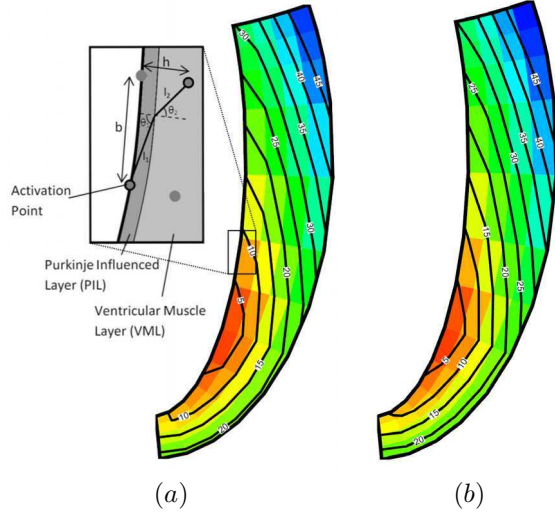


Figure 2. (a) Activation time map and the configuration of the computational domain in this model and (b) Activation time map from Eikonal model.

3. Results

A simple left ventricular geometry was used to test the performance of the model. The model is constructed using prolate-spheroidal coordinates following the method described in [11] to build a normal canine ventricular geometry. In order to illustrate the wavefront propagation in the domain, the action potential is calculated in 65 regularly distributed control points on the axi-symmetric surface of the model (see Figure 2). The geometrical parameters b and h can be easily calculated by integrating over the trace from the projection of the target point to the stimulation and target points respectively as below,

$$h, b = \int_0^{t_a} \sqrt{x(t)_t^2 + y(t)_t^2} dt. \quad (10)$$

In order to model the conduction velocity map parameters, we used a two-domain geometry with a thin, only Purkinje Influenced Layer (PIL) of thickness $1.5mm$ and a normal Ventricular Muscle Layer (VML). Here, the conductivity is assumed isotropic inside each layer, i.e. $k = 1$. The values of conduction velocity in the PIL and VML are set to 1.59 and $0.49 \frac{m}{s}$ respectively [9]. Having the geometrical parameters and velocity map, the activation times for the control points are calculated from algorithm I.

Figure 2 (a) shows the resulting activation time distribution in the model. We compared the result of our model with the Eikonal model for the same geometrical configuration. We used the diffusion-free form of Eikonal equation that could be discretized as below,

$$\Delta t = \frac{1}{v} \sqrt{\mathbf{x}(\mathbf{FDF}^T)^{-1}\mathbf{x}^T}. \quad (11)$$

here \mathbf{x} is the position vector, \mathbf{F} the fiber direction matrix and \bar{D} the anisotropic ratio of space constants respect to the fiber direction. The homogeneous Eikonal equation is solved in the domain using the Fast Marching Method (FMM) [12]. For this mesh-based approach, it is necessary to find the optimum mesh size that provides independency of the results from the mesh resolution. We found that for meshes with more than 30401 nodes the results were consistent. The activation sequence obtained from the Eikonal solver is shown in Figure 2 (b). Since there is not a standard for comparing the results from the Eikonal and current model, we used the rate of change of the values at the control points as the characteristic factor. Then for a given mesh size, a Mean Relative Difference (MRD) factor was defined as,

$$MRD = \frac{1}{M} \sum_{m=1}^M \frac{|t_{a,m}^{Eikonal} - t_{a,m}^{MultiLayer}|}{t_{a,m}^{MultiLayer}} \times 100 \quad (12)$$

where $t_{a,m}$ is the activation time for the control point m , and M is the total number of the control points. Figure 3 provides the relation of the MRD factor with the number of nodes in the Eikonal method. This figure shows that by increasing the number of computational nodes, MRD factor continuously descending. It can be concluded that at very refined computational domain and very high number of nodes, the results from Eikonal method tend to the results from the current approach. Since the rate of change always decreases, we conclude that the current method provides results that are more accurate than the mesh based Eikonal solution. It is expected that due to the closed-form estimation of the signal path in our method, its estimation is more accurate than mesh-based approaches. It is also expected that due to the mesh free features, less computational time is required for the proposed model. We checked the computational time that is needed to calculate the activation time for 65 control nodes. With the Eikonal method it took 22.5 seconds to perform the computations, whereas for our method the required time was about 1.4 seconds. The difference is due to the mesh based feature of the Eikonal

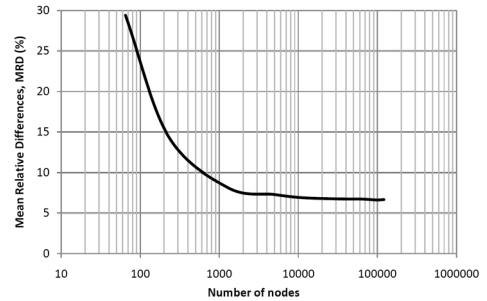


Figure 3. Mean Relative Difference (MRD) as a function of computational node number in Eikonal model.

method in which computational time covers the unnecessary nodes so that it depends on the number of computational nodes. On the other hand, the current method is a point based approach that does not require to calculate in the entire nodal domain. The calculations are performed with MathWorks Matlab software.

4. Discussion and conclusions

A mesh-less approach for calculation of activation time in the ventricular wall has been presented. This formulation has been adapted for a multi-layer configuration of the ventricular wall, and it has been evaluated in a two layer domain. It models the effect of purkinje distribution and transmural change in myofiber conductivity. Comparison with the diffusion-free Eikonal method for the simplified two layers and homogenous regions shows that there is a good match between the results of the Eikonal and current approach. Also calculations with different mesh sizes show that with the Eikonal method, an increase of the number of nodes decreases the difference between the results of both methods. Therefore, for coarse meshes the method presented could be considered as more appropriate and accurate than the Eikonal model. The mesh-free feature makes it suitable for the mechanical simulations with large hexahedral elements in which a fast calculation of electrophysiological activation time at an arbitrary point and without construction of meshes for electrical part is required. We used a single source stimulus in the model, but the method could be applied to scenarios with several sources by simply calculating the time of approach from each of the points and choosing the minimum action time. This model can potentially useful in applications for real-time computation of electrical activation time.

Despite of its rapidity and simplicity, the method suffers the lack of the detailed action potential propagation information such as the repolarization stages that means some functional abnormalities such as fibrillation cannot be studied. This approach is not able to model the diffusion feature of action potential propagation in the myocardium. It do not take the myofiber longitudinal and traversal conductivities into account.

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