Transform Based Approach for ECG Period Normalization

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Abstract

The irregularity in the ECG heartbeats durations and application of two dimensional ECG compression algorithms has undoubtedly been a challenge in this field. In this paper, an efficient alternative solution for ECG period normalization is proposed. Each ECG heartbeat is transformed into the SVD domain formed from the LPC filter impulse response matrix, where only a few components contain most of the energy of the signal. The transformed signal is zero padded or truncated to match the desired length then, multiplied by a basis of higher or lower dimension, respectively, to form a normalized ECG heartbeat. Reverse steps are applied to recover the heartbeat with original length. It is proven that an exact recovering of the original signal is achieved when it is stretched. On the other hand, the limit in shrinking the signal without producing significant distortion is also analysed using objective measure of distortion. In addition, it is shown that the singular vectors are orthogonal sinusoids which lead to a reduction of the computational complexity of the algorithm.

1. Introduction

The electrocardiogram is an important physiological signal for the identification of abnormal cardiac rhythms. In recent years, extensive research has been devoted to developing two-dimensional ECG compression algorithms, based on the need for an efficient storage of long-term recordings. The essence is to exploit further the redundancy in the signal as compared to one-dimensional algorithms and thus yield a better compression ratio. The irregularity in the ECG heartbeats durations has undoubtedly been a challenge in this field, where different techniques have been proposed to equalize the heartbeats to the same period length. These techniques can be classified into two main categories, namely signal extension, and period normalization techniques. [1]. The proposed algorithm can be classified under the latter category.

In the signal extension category the original segment is padded by a suitable number of samples to match with the maximum period length, including zero padding, zero-order extension, and mean extension. Alternatively, a widely used technique for period normalization was reported by [2] and applied to the ECG by Wei et al [3]. However, this does not perform well when the ECG is extremely irregular [1]. Other solutions such as interpolation have been proposed by other authors [4]. Although period normalization techniques are lossy when compared to the signal extension techniques, a better percent root mean square difference (PRD) is achieved [1].This is justified by the fact that the period normalization provides higher inter-beat correlations when compared to signal extension. Two objectives are to be achieved when developing any period normalization technique:

1- The distortion due to the normalization should be minimized.

2. The morphology of the original signal should be preserved when stretched or shrunk.

In this paper, an efficient alternative solution for period normalization is proposed. Using the proposed technique, the original signal is recovered losslessly when stretched. Also, the distortion due to the shrinking of the signal is minimised due to the optimality of the SVD. An analysis is carried out to define the limit in shrinking the signal without producing significant distortion.

2. Mathematical formulation

A first order linear predictive analysis expressed in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{He}\ , \tag{1}$$

is applied to an ECG signal , where $\boldsymbol{Y}$ and $\boldsymbol{e}$ are $N \times 1$ column vectors their entries represented by the ECG samples and the residual error respectively. The entries of the impulse response matrix $\boldsymbol{H}$ are completely determined by the linear prediction coefficients, $\boldsymbol{H}$ is lower triangular and Toeplitz.

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & \cdots & \cdot & 0 \\ h(1) & 1 & \ddots & \cdot & 0 \\ \vdots & \vdots & \ddots & \cdot & \vdots \\ . & . & . & \cdot & . \\ h(N-1) & h(N-2) & . & \cdot & 1 \end{bmatrix} \tag{2}$$

Applying the singular values decomposition (SVD) to $\boldsymbol{H}$ gives:

$$Y = UDV^T e \quad , \tag{3}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal $N \times N$ matrices, and $\boldsymbol{D}$ is a real $N \times N$ diagonal matrix of the singular values .

The SVD domain representations of $\boldsymbol{Y}$ and $\boldsymbol{e}$ are given by: $\boldsymbol{\theta}$ and $\boldsymbol{\zeta}$ respectively, where $\boldsymbol{\theta} = \boldsymbol{U}^T\boldsymbol{Y}$ and $\boldsymbol{\zeta} = \boldsymbol{V}^T\boldsymbol{e}$ [5].

Therefore:

$$\boldsymbol{\theta} = \boldsymbol{D}\boldsymbol{\zeta} \tag{4}$$

From equation (4) each component of the residual signal is projected onto the right eigenvectors of the matrix $\boldsymbol{H}$ and then multiplied by the corresponding singular value. Since the singular values are always arranged in a descending order [6], one can expect that the transformed ECG signal ($\boldsymbol{\theta}$) is decaying.

3. Period normalization

3.1. Approach

The schematic block in Figure.1 summarizes the proposed period normalization technique. A first order linear prediction is applied to the ECG heartbeats. Each heartbeat is then expressed as a linear combination of the left singular vectors of the LPC filter's impulse response matrix. The components of transformed ECG vector in the directions of the last singular eigenvectors are very small or equal to zeros, Figure.2 (a). Hence, by truncating these components, it is expected that there would be no loss of information; but only a reduction in the noise level present in the signal. On the other hand, when the vector is zero padded its energy is preserved. The resulting vector is multiplied by the left singular vectors matrix of the desired size ($\boldsymbol{U1}_{N^*\times N^*}$) to form a normalized ECG segment. Reverse steps are applied to recover the heartbeats with its original lengths. One may choose to retain the LPC coefficient as side information to be used to generate the exact bases during the recovering stage, or to instead calculate a new coefficient from the stretched or shrunk signal, respectively. Only the first choice has been considered in this paper.
It can be proven that the energy of the original and recovered signal are the same when subjected to stretching. Consequently, a noise-free recovering is achieved as shown in Figure.2 (b).

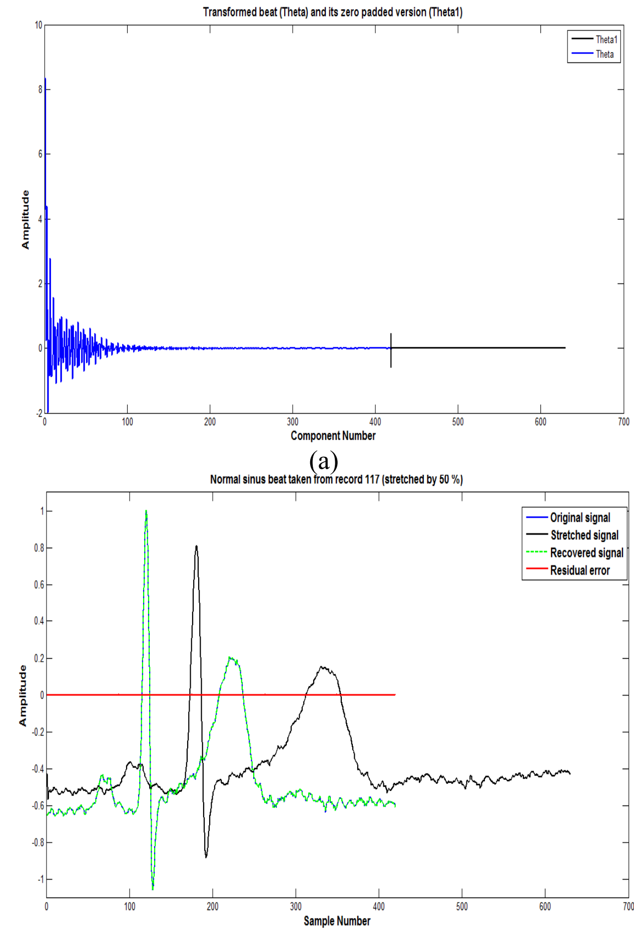

Figure 2. a-Transformed ECG ($\boldsymbol{\theta}$), b- Distortion-free recovering of original signal taken from record 117 MIT-BIH arrhythmia database (stretched by 50%).

Proof:

1. The mapping $\boldsymbol{U}^T\boldsymbol{Y}$ is isometric:

$$\|\boldsymbol{\theta}\|^2 = \|\boldsymbol{U}^T\boldsymbol{Y}\|^2 = (\boldsymbol{U}^T\boldsymbol{Y})^T(\boldsymbol{U}^T\boldsymbol{Y}) = \boldsymbol{Y}^T\boldsymbol{U}\boldsymbol{U}^T\boldsymbol{Y}$$
$$= \boldsymbol{Y}^T\boldsymbol{Y} = \|\boldsymbol{Y}\|^2 \ , (\boldsymbol{U}\boldsymbol{U}^T = \boldsymbol{I}), \boldsymbol{U} \text{ is orthonormal.}$$

2. Zero padding the transformed ECG ($\boldsymbol{\theta}$) does not change its norm:

$$\|\boldsymbol{\theta 1}\| = \|\boldsymbol{\theta}\|.$$

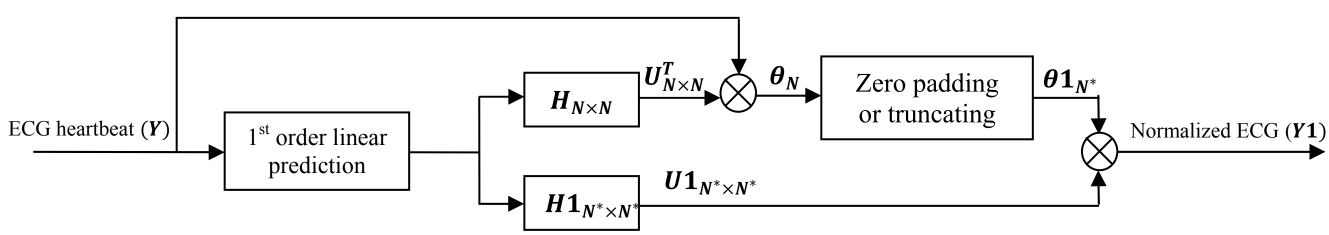

Figure 1. Schematic block diagram of the proposed period normalization technique.

3.The mapping $\boldsymbol{U1\theta1}$ is isometric.

$$\begin{aligned}\|\boldsymbol{Y1}\|^2 = \|\boldsymbol{U1\theta1}\|^2 &= (\boldsymbol{U1\theta1})^{T}(\boldsymbol{U1\theta1}) \\ &= \boldsymbol{\theta1}^{T}\boldsymbol{U1}^{T}\boldsymbol{U1\theta1} \\ &= \boldsymbol{\theta1}^{T}\boldsymbol{\theta1} = \|\boldsymbol{\theta1}\|^2 \\ = \|\boldsymbol{\theta}\|^2 = \|\boldsymbol{Y}\|^2 \quad .\end{aligned}$$

4. The reverse steps are isometric (See Fig.3 : all the transitions are isometric).

$$\begin{tikzcd} \boldsymbol{Y_N} \arrow[r, bend left, "\boldsymbol{U^T_{N\times N}}"] & \boldsymbol{\theta_N} \arrow[l, bend left, "\boldsymbol{U_{N\times N}}"] \arrow[r, bend left, "\text{Zero-padding}"] & \boldsymbol{\theta1_{N^*}} \arrow[l, bend left, "\text{Truncating zeros}"] \arrow[r, bend left, "\boldsymbol{U1_{N^*\times N^*}}"] & \boldsymbol{Y1_{N^*}} \arrow[l, bend left, "\boldsymbol{U1^T}_{\boldsymbol{N^*\times N^*}}"] \end{tikzcd}$$

Figure 3. The proof in a graphical form

One can decide to preserve 95% of the energy of the signal while shrinking it, or by pre-defining the PRD1 % allowed and then determining the number of coefficients that can be truncated.

3.2. Performance measurement

Two approaches can be used to evaluate the performance of the recovered signal in case of shrinking. The first is based on subjective measure of distortion via visual inspection. The second uses objective measures. The percent root mean square difference independent of the mean value (PRD1%) has been used to evaluate the error between the original and reconstructed waveforms [7]:

$$PRD1\% = \sqrt{\frac{(\boldsymbol{Y}-\widehat{\boldsymbol{Y}})^{T}(\boldsymbol{Y}-\widehat{\boldsymbol{Y}})}{(\boldsymbol{Y}-\overline{\boldsymbol{Y}})^{T}(\boldsymbol{Y}-\overline{\boldsymbol{Y}})}} \times 100 \qquad (5)$$

where: $\boldsymbol{Y}$ is the original signal, $\widehat{\boldsymbol{Y}}$ and $\overline{\boldsymbol{Y}}$ are respectively the reconstructed signal and the mean value of the original signal.

Figure.4 illustrates the effect of truncation of $\boldsymbol{\theta}$ (when the signal is to be shrunk) on the PRD1 between a normal beat taken from record 117, Figure.1 (b), and the recovered one. The graph can be segmented into three parts. In the first, the effect of truncation on the recovered ECG is very small. In the second, the PRD1 started to increase gradually until the last part where the distortion became significant.
Figure 5 illustrates the effect of shrinking on an ectopic beat taken from record 119.

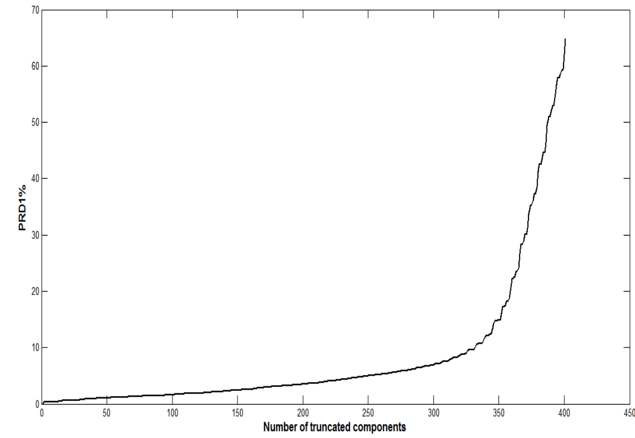

Figure 4. Example of the change of the PRD1 with the number of truncated coefficients (record Figure 2).

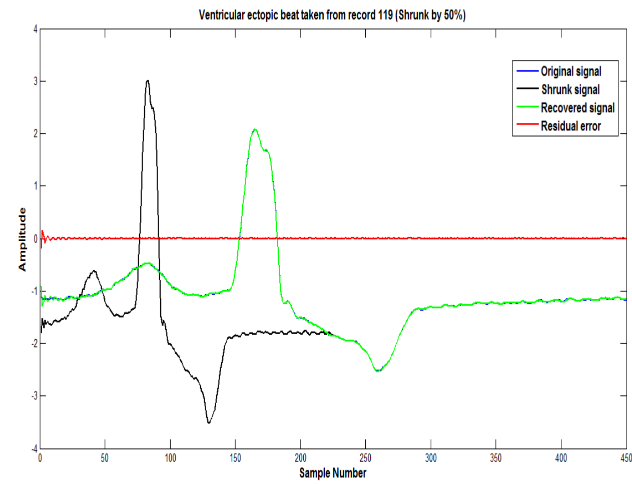

Figure 5. Nearly distortion-free recovering of original signal taken from record 119 MIT-BIH arrhythmia database (shrunk by 50%).

4. The left singular vectors

Though the proposed pre-processing technique is for off-line algorithms, yet reduction of computational complexity is worthwhile. The choice of first order linear prediction is justified by the fact that in this case the left singular vectors of the matrices $\boldsymbol{H}$ and $\boldsymbol{H1}$ are orthogonal sinusoids. It is important to mention that for practical applications, only the matrix $\boldsymbol{U}$ is needed.
In the case of first order linear prediction the matrix $\boldsymbol{H}$ has the form:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & \cdots & \cdot & 0 \\ a & 1 & \cdots & \vdots & . \\ a^2 & a & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & 1 & 0 \\ a^{N-1} & a^{N-2} & & a & 1 \end{bmatrix}, \qquad (6)$$

where : $a = -a_1$, a_1 is the LPC coefficient . Note that magnitude of a is less than 1, since $a = \frac{R1}{R0}$ and $|R1| < |R0|$. $R0$ and $R1$ are autocorrelation coefficients of the signal [8].
Its inverse $\boldsymbol{B}$ is bi-diagonal with 1 on the main diagonal and the LPC coefficient on the diagonal below [9].

$$\boldsymbol{B} = \boldsymbol{H}^{-1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -a & 1 & . & . & 0 \\ 0 & -a & \vdots & \vdots & \vdots \\ \vdots & \vdots & . & . & 0 \\ 0 & 0 & . & -a & 1 \end{bmatrix} \qquad (7)$$

The ordinary inverse of $\boldsymbol{H}$ is obtained by inverting its SVD factors (Equation 3) and writing them in reverse order [10]:

$$\boldsymbol{B} = \boldsymbol{V}\boldsymbol{D}^{-1}\boldsymbol{U}^{\boldsymbol{T}} \qquad (8)$$

$\boldsymbol{U}$ is the right singular vector matrix of $\boldsymbol{B}$ and the eigenvector matrix of the $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B}$. $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B}$ is symmetric, uniform and tridiagonal:

$$\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B} = \begin{bmatrix} a^2+1 & -a & 0 & \cdots & 0 \\ -a & a^2+1 & -a & \ddots & 0 \\ 0 & -a & . & . & . \\ \vdots & \vdots & . & a^2+1 & -a \\ 0 & 0 & \cdots & -a & 1 \end{bmatrix} \qquad (9)$$

The problem is brought forward to find the eigenvectors of the matrix $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B}$ which are given by: $u_{jk} = A \sin j\phi_k$, A is chosen so that the norm of u_k is unity.

ϕ_k, $k = 1,\ldots, N$, are the solutions of the equation:

$$a \sin N\phi - \sin\,(N+1)\phi = 0 \qquad (10)$$

Example of the singular vectors of the matrices $\boldsymbol{H}$ and $\boldsymbol{H1}$ are shown in Figure 6.

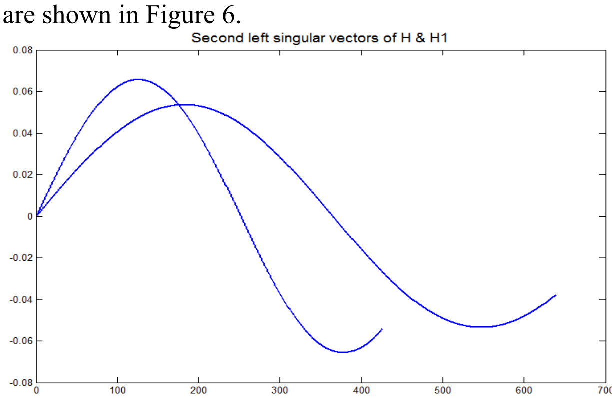

Figure 6. Example of left singular vectors of $\boldsymbol{H}$ and $\boldsymbol{H1}$.

5. Discussion and conclusions

In addition to developing a period normalization technique that is able to preserve the morphology and the energy of the ECG signal, the computational complexity of the proposed algorithm was reduced as well.
With further research, other alternative solutions for period normalization could be investigated, for instance:
1. Most times we found that the LPC coefficient was close to unity in value. Here, it is suggested that it could be replaced by one, which will lead to a much simpler implementation with a slightly reduced performance.
2. As mentioned earlier, the preservation of the LPC coefficient as side information could be dropped, and a new coefficient could be estimated from the stretched/shrunk signal respectively and used to compute the singular vectors.

Acknowledgements

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References

[1] Chou HH, et al. An Effective and Efficient Compression Algorithm for ECG Signals with Irregular Periods. IEEE Trans Biomed Eng 2006; 53 (6): 1198-1205.
[2] Kanjilal PP , Palit S. On multiple pattern extraction using singular value decomposition. IEEE Trans. Signal Processing 1995; 43 :1536-1540.
[3] Wei J, et al. ECG data compression using truncated singular value decomposition. IEEE Trans Inf. Tech. Biomed 2001; 5 : 290-299.
[4] Ramakrishnan AG, Saha S. ECG Coding by Wavelet-Based Linear Prediction. IEEE Trans. Biomed.Eng 1997 ; 44 (12): 1253-1261.
[5] Atal B .A model of LPC excitation in terms of eigenvectors of the autocorrelation of the impulse response of the LPC filter. ICASSP 1989;1 :45-48.
[6] Golub GH, Van Loan CF, Matrix Computation, 3rd ed. Baltimore: Johns Hopkins University Press, 1989.
[7] Blanco-Velasco M,et al.On the use of PRD and CR parameters for ECG compression. Medical Engineering & Physics 2005; 27 : 798-802.
[8] Makhoul J. Linear prediction: A tutorial review. Proc. IEEE 1975; 63 (4): 561-580, Apr. 1975.
[9] Strang G, Nguyen TQ.Wavelets and Filter Banks. Cambridge, MA: Wellesley-Cambridge, 1996.
[10] Kalman D. A Singularly Valuable Decomposition: The SVD of a Matrix. College Math 1996; 27(1): 2-23.

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