Fractal Dimension of Mean Arterial Pressure and Heart-Rate Time Series from Ambulatory Blood Pressure Monitoring Devices

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Abstract

Ambulatory blood pressure monitoring (ABPM) devices provide 24-hour profiles of mean arterial pressure (MAP) and heart rate (HR) by inflating an arm-cuff every 15 minutes during daytime and every 20 minutes during night-time. Aim of this work is to evaluate whether the intrinsic structure of ABPM dynamics can be described during day and night subperiods by calculating the fractal dimension (FD) of MAP and HR.

For this aim, first we evaluated the performances a recently proposed FD estimator on short segments of fractional Brownian motions. Then we applied the new FD estimator on 24-hour ABPM recordings of two healthy volunteers. The FD estimator showed good performances on synthesized data, with lower bias compared to Higuchi's method. FD estimates of real data tended to be higher during the day. In particular, FD of daytime MAP (1.84 ±0.06 M±SD) was substantially higher than 1.5, suggesting that anticorrelation mechanisms may influence the diurnal long-term regulation of blood pressure

1. Introduction

Ambulatory blood pressure monitoring (ABPM) devices are more and more used to provide profiles of mean arterial pressure (MAP), systolic and diastolic blood pressure (BP), and heart rate (HR) over the 24 hours in freely moving subjects. ABPM devices measure arterial BP by inflating and slowly deflating an arm cuff. Auscultatory ABPM devices identify systolic and diastolic BP directly by detecting the Korotkoff sounds with a microphone, and derive MAP indirectly from systolic and diastolic BP values. Oscillometric ABPM devices measure MAP directly from vibrations produced by the arterial walls and calculates systolic and diastolic BP through specific algorithms. In any case, the frequency at which the measures are performed cannot be too high in order to avoid interfering with patients activities or compromising the quality of sleep. For these reasons, measures are usually performed every 15 minutes during day-time and every 20 minutes at night.

The clinical value of ABPM devices consists in their ability to recognize white coat hypertension, masked hypertension or alterations in dipping patterns at night from BP levels calculated over standardised day-time and night-time subperiods (1-3). The availability of different measures over day-time (usually 48 measures between 10 AM and 10 PM) and night-time (usually 18 measures between 0 AM and 6 AM) might also allow deriving useful information from BP and HR variability during the day and the night. Till now, however, the only parameters describing the BP dynamics from ABPM devices are the amplitudes of BP and HR oscillations as quantified by day-time and night-time standard deviations, taken separately or combined in a single weighted average (4). No measures of the intrinsic structure of BP or HR dynamics during day-time or night-time are actually derived from ABPM devices.

Aim of the present study is to evaluate the feasibility of the estimation of a specific aspect of BP and HR dynamics related not to the amplitude of the fluctuations but to the degree of long-range dependence and "convolutedness" of the time series: the fractal dimension, FD.

Algorithms for estimating FD usually requires a large number of samples. By contrast, time series from ABPM devices are relatively short due to their low sampling rate. Therefore the first part of this work makes use of synthesised series and evaluates the performances of a recently proposed algorithm for estimating FD which seems suitable for analysing relatively short time series (4).

The second part of the study applies the same FD algorithm to ABPM recordings obtained in two healthy volunteers in order to provide the first description of BP and HR "convolutedness" during day and night in terms of fractal dimension.

2. FD estimation of short time series

This section evaluates the performances of a recently proposed algorithm (corrected fractal dimension, FD_C) for estimating the FD of short time series (5). The method is evaluated by analysing segments of synthesized data with known theoretical FD. The length of simulated segments is the same of ABPM data obtainable during daytime or night-time sub-periods. Performances are compared with those of the Higuchi's algorithm, FD_H.

2.1. Corrected fractal dimension, FD_C.

The algorithm is based on the correction of a fundamental flaw in the popular Katz's method (6). The Katz's method was inspired by the Mandelbrot's suggestion that the fractal dimension of a river can be calculated from the river length, L, and the distance between source and mouth, d, as:

$$\text{FD}=\log(L)/\log(d) \qquad (1).$$

In order to calculate the fractal dimension of a waveform composed by n points $\{y_i\}$ measured at times $\{t_i\}$, with $1 \le i \le n$, Katz proposed to plot a bidimensional curve in the ***Y-T*** space defined by points of coordinates (t_i, y_i), and to apply eq.(1) to this curve. The Euclidean distance between two points, i and j, of the curve was defined as:

$$l_{i,j}=\sqrt{\left(y_i-y_j\right)^2+\left(t_i-t_j\right)^2} \qquad (2)$$

In this way, length L and extension d of the curve were:

$$L=\sum_{i=1}^{n-1} l_{i,i+1} \qquad (3)$$

$$d=\max\left\{l_{i,j}\right\} \qquad (4)$$

By normalizing d and L by the length of the average step, $L/(n-1)$, eq.(1) becomes:

$$\text{FD}=\log(n-1)\Big/\left[\log(n-1)+\log\left(d/L\right)\right] \qquad (5).$$

The critical point of this approach is that t_i and y_i are intrinsically different quantities, being one the time index and the other the measured quantity. For this reason, the distance in eq.2 is not well defined. To overcome this problem, it has been proposed to calculate eq. (5) directly on the mono-dimensional space ***Y*** defined by the n coordinates $\{y_i\}$ (5). In this way length L and extension d of the curve are:

$$L=\sum_{i=1}^{n-1}\left|y_{i+1}-y_i\right| \qquad (6)$$

$$d=\max\{y_i\}-\min\{y_i\} \qquad (7)$$

It should be considered, however, that the direct application of (6) and (7) in (1) may lead to overestimating the true FD when data are strongly anti-correlated. In this case the curve tends to "retrace its steps": therefore d may tend to an asymptotic value while L increases indefinitely. For instance, this happens when the curve describes a periodic trajectory every P samples, like a sinusoid of amplitude A and period P. In this case d reaches its maximum value, A, after one period P while L continues to increase with n, leading FD to rise logarithmically with n. To avoid this overestimation, the proposed method calculates L and d over the subset of the n points for which the extension d is half the value measured for the whole dataset.

The procedure is summarized by the following steps:

1. d is calculated from the whole dataset as in eq.(7);
2. the dataset is scanned to identify the average size n_W with extension at least equal to $d/2$ (in any case n_W should not be lower than 8 samples for statistical consistency);
3. the dataset is split into M consecutive, overlapped windows of n_W points, and the fractal dimension FD_i is calculated in each window "i" by eq.(5)-(7);
4. FD of the whole dataset is estimated as the median value of FD_i ($1 \le i \le M$).

Let's call "corrected" FD, i.e., FD_C, this estimate obtained by correcting the original Katz's method.

2.2. Higuchi's fractal dimension, FD_H.

Another popular method for assessing the fractal dimension of a time series is due to Higuchi (7). Given the series $\{y_i\}$ of N points, first an interval time k is selected and k time series are constructed from $\{y_i\}$ as:

$$y_m, y_{m+k}, y_{m+2k}, \ldots, y_{m+Pk}$$

with P the integer part of the fraction $(N-i)/k$ and $1 \le m \le k$. The length of each curve m is:

$$L_m(k)=\sum_{j=1}^{P}\left|y_{m+jk}-y_{m+(j-1)k}\right|\frac{N-1}{P\cdot k^2} \qquad (7)$$

The average length for all the m curves generated by the interval k is:

$$L(k)=\frac{1}{k}\sum_{j=1}^{k}L_j(k) \qquad (8).$$

For a fractal time series, $L(k) \propto k^{-\text{FD}}$ and FD can be estimated as slope of the regression line between $L(k)$ and k in a log-log scale. Therefore the Higuchi's method calculates $L(k)$ for k between 1 and k_{MAX}. Usually k_{MAX} is set equal to 5. Let's call FD_H the estimation of FD by the Higuchi's method. In this work, FD_H has been estimated by means of the code provided in (8).

2.3. Synthesized data

The FD_C and FD_H algorithms were tested by synthesising segments of fractional Brownian motion with theoretical FD between 1 and 2. The Matlab function *wfbm(H,n)* with H the Hurst exponent, was used to synthesize 100 time series for each fractal dimension

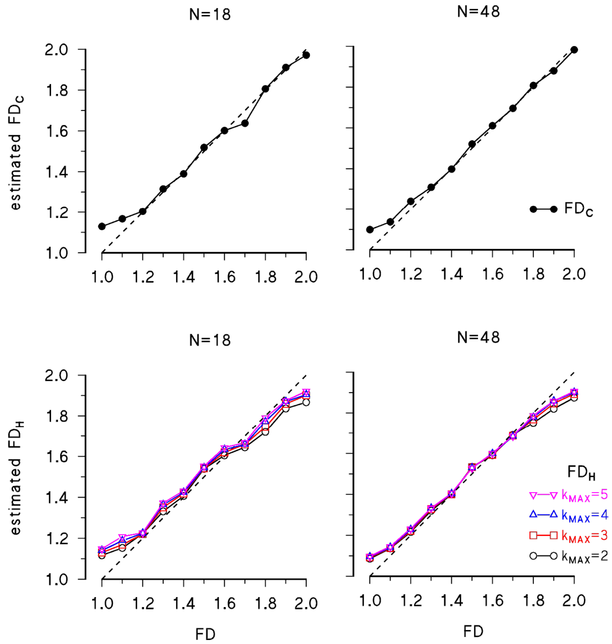

Figure 1. Mean values of FD_C (upper panels) and FD_H (lower panels) for segments of fractional Brownian motions with theoretical FD between 1 and 2 and length N equal to 18 or 48 samples.

FD=2-H and for sample sizes N=18 and N=48. These sample sizes correspond to the same length of ABPM time series recorded during night-time or daytime sub-periods.

3. Simulation results

Figure 1 shows mean values of FD_C and FD_H estimates obtained for data segments synthesised at different theoretical FD values. As expected, estimates are closer to the theoretical value (dashed line) when longer segments are analysed (N=48). Both methods show a similar positive bias when the true FD is lower than 1.2. However, at difference from FD_C, FD_H also shows negative bias, underestimating the theoretical value when FD is greater than 1.8. No substantial differences appear in FD_H when k_{MAX} decreases from k_{MAX}=5, value traditionally selected in most studies, up to k_{MAX}=2.

The absolute error, as percentage of the theoretical FD, is reported in table 1 for FD_C and for FD_H, this latter estimated with k_{MAX}=5. FD_C provides lower estimation bias over almost the whole range of FD values.

Table 1. Absolute error of FD_C and FD_H estimates as percentage of the theoretical FD value for segments of synthetised fractional Brownian motion of length equal to 18 and 48 samples.

	N=18		N=48	
FD	FD_C	FD_H	FD_C	FD_H
1.0	12.9%	14.8%	9.9%	9.8%
1.1	6.1%	9.9%	3.3%	4.1%
1.2	0.3%	2.3%	3.3%	2.7%
1.3	1.2%	5.6%	0.7%	2.5%
1.4	0.8%	2.3%	0.1%	0.4%
1.5	1.2%	3.5%	1.4%	1.9%
1.6	0.1%	2.7%	0.7%	0.1%
1.7	3.7%	1.9%	0.2%	0.9%
1.8	0.3%	0.5%	0.4%	0.7%
1.9	0.6%	1.3%	1.0%	2.2%
2.0	1.4%	3.9%	0.8%	4.7%

FD_H estimated with k_{MAX}=5

4. Application on real data

4.1. Subjects and methods

The FD_C estimator was applied on real ABPM data recorded on two healthy volunteers (females, age 41 and 29 years). The 24-hour ABPM recordings were performed with an oscillometric device (AND TM2430, A&D Company, Tokyo, Japan). Recordings were performed twice, at about one month interval, in each subject.

Figure 2 shows an example of MAP and HR series over the 24 hours. The figure also indicates the position of day-time (48 measures) and night-time (18 measures) sub-periods selected for FD analysis.

Mean value, standard deviation (SD) and FD_C of MAP and HR were calculated in each recording separately over day-time and night-time. Measures repeated in the same subject at one-month interval were averaged.

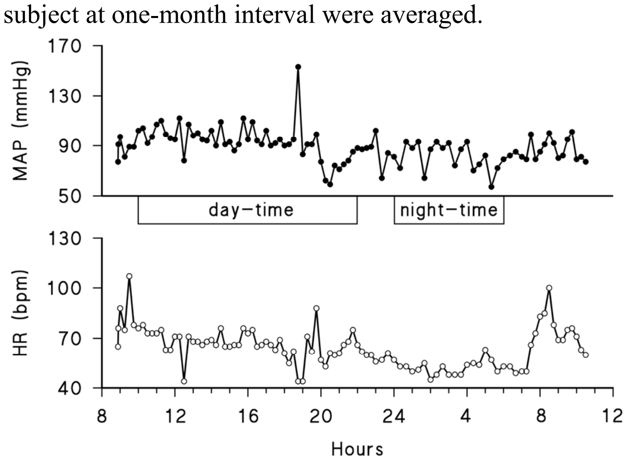

Figure 2. Example of ABPM values of MAP and HR measured in one volunteer over the 24 hours, with selected day-time and night-time subperiods.

4.2. Results

In both of our volunteers, MAP and HR mean levels decreased from daytime to night-time subperiods. Mean values ±SD were: 90.2 ±6.1 vs. 73.7 ±2.1 mmHg for MAP; 72.9 ±7.5 vs. 61.8 ±10.0 bpm for HR. Also SD of

MAP and HR decreased from daytime to nighttime in both subjects. Individual values are shown in figure 3 (lower panels). On average, SD decreased from 10.8 ±1.5 to 7.6 ±1.6 mmHg for MAP, and from 11.2 ±3.6 to 5.5 ±0.6 bpm for HR.

Table 2. FD_C estimates (mean ±SD)

	Day-time	Night-time
MAP	1.84 ±0.06	1.59 ±0.27
HR	1.63 ±0.10	1.48 ±0.23

Also FD_C estimates tended to decrease from day-time to night-time (see table 2). However, at differences from SD changes, which decreased consistently in both subjects, night-day changes of FD_C were not homogeneous. In fact, FD_C decreased substantially from day to night in both MAP and HR, falling to values lower than 1.5 at night, in one of the volunteers while in the other volunteer FD_C decreased only slightly for MAP, and even increased slightly for HR, remaining higher than 1.5 in both sub-periods.

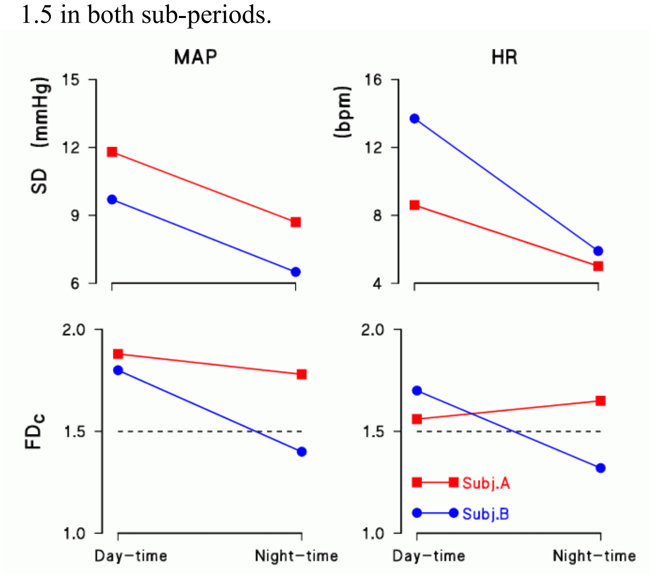

Figure 3. Estimates of FD and SD of MAP and HR during daytime and night-time subperiods in two volunteers.

5. Comments and conclusions

Aim of this work was to evaluate the feasibility of FD assessment in MAP and HR series recorded with ABPM devices. Given the low sampling rate of these devices, and the resulting short length of night-time and day-time series, our main concern regarded the availability of valid algorithms for FD estimation of very short series. The analysis with synthesized data showed that the recently proposed FD_C method can provide FD estimates with negligible bias even with data segments of 18 samples only. In particular, the FD_C estimator has lower bias compared to the classic Higuchi's method when high FDs are considered. This property is useful given the high FD estimates obtained for real MAP data.

The application of the method on ABPM data in two healthy subjects only does not allow us deriving statistically significant conclusions on night-day modulations of FD. However, preliminary data suggest some interesting aspects of long-term ABPM dynamics.

First, FD changes between day and night do not follow the substantial changes observed in SD of MAP and HR. This would suggest that FD may actually represent aspects of long-term cardiovascular control that are not described by day-night modulations of SD.

Second, FD_C values of MAP appear remarkably high during daytime. In particular, they appear substantially greater than 1.5, the FD of a pure Brownian motion. This high FD value may suggest the presence of anticorrelation mechanisms influencing the diurnal long-term regulation of blood pressure.

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