

# Improving the Accuracy of Forward Computations: Different Methods to Implement the Propagation of the Depolarization Wave Front

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## Abstract

*This study addresses the problem of discontinuities in the body surface potentials (BSPs) introduced by the discrete nature of a cardiac surface source model. These physiologically unrealistic discontinuities result from the switching on of the elementary surface source elements based on the timing of activation assigned to their nodes. Two new methods for avoiding such discontinuities are introduced and their effectiveness is analyzed.*

*In the first method only the contributions of the activated parts of the source elements are evaluated. This results in a more physiologically realistic description of cardiac source activity and is referred to as the gold standard. In addition, a faster, approximate method was developed, in which the contribution of any source element at a particular time is weighted with the activated fraction of its surface.*

*Both methods prevent the discontinuities; the fast method provides a useful approximation and yields a fast implementation as required in interactive forward simulations and is useful in inverse modeling.*

## 1. Introduction

This paper addresses the problem of discontinuities in the BSPs that is introduced by the discrete nature of the cardiac surface source model in electrographic source imaging [1,2]. These physiologically unrealistic discontinuities result from the switching on of entire source elements based on the timing of depolarization assigned to the nodes specifying their geometry. They are most prominent in high amplitude signals such as observed in the precordial leads.

In our previous applications of this source model these artifacts were reduced by the application of a low-pass moving average filter [2,3]. However, in some situations this yields unsatisfactory results, in particular if the artifacts overlap with real physiological short-term fluctuations in the BSPs. Moreover, the amplitude of the discontinuities depends strongly on the difference in activation times between the nodes, and hence on the

activation pattern and mesh density, which makes it impossible to determine a filter setting that is adequate for all situations while not affecting the ECG wave forms.

In this paper we introduce and compare two new methods aimed at avoiding such discontinuities without filtering.

## 2. Methods

The source model addressed is the equivalent double layer model (EDL). It represents the electric currents generated by all cardiac myocytes by means of an equivalent double layer at the surface  $S_m$  bounding the myocardium. Its strength,  $S(\vec{x}, t)$ , is proportional to the transmembrane potential (TMP) at  $\vec{x}$  [4-6]. The potential  $V(\vec{y}, t)$  at any observation point  $\vec{y}$ , and time  $t$ , such as those on the body surface, follows from

$$V(\vec{y}, t) = \int_{S_m} A(\vec{x}, \vec{y}) S(\vec{x}, t) dS_m, \quad (1)$$

where  $A(\vec{x}, \vec{y})$  is the potential generated at  $\vec{y}$  by a unit source element at  $\vec{x}$ .

The discretized version of equation (1) reads

$$V_i(t) = \sum_j \tilde{a}_{ij} S_j(t), \quad (2)$$

with  $i$  any observation point,  $j$  any node of the discretized version of  $S_v$  and  $\tilde{a}_{ij}$  denoting the transfer based on the volume conduction effects in the medium surrounding the source.

In our work, the potentials are evaluated by using the Boundary Element Method (BEM) [3], in which both  $S_m$  and the surfaces specifying any inhomogeneity in the volume conduction are represented by numerous, small planar triangles. The strength over each triangle of the BEM configuration is expressed as a linear function of the values at their nodes (vertices) [7]. Accordingly, the double layer strength  $S(\vec{x}, t)$  over each source triangle is

$$S(\vec{x}, t) = g_1(\vec{x})S_1(t) + g_2(\vec{x})S_2(t) + g_3(\vec{x})S_3(t), \quad (3)$$

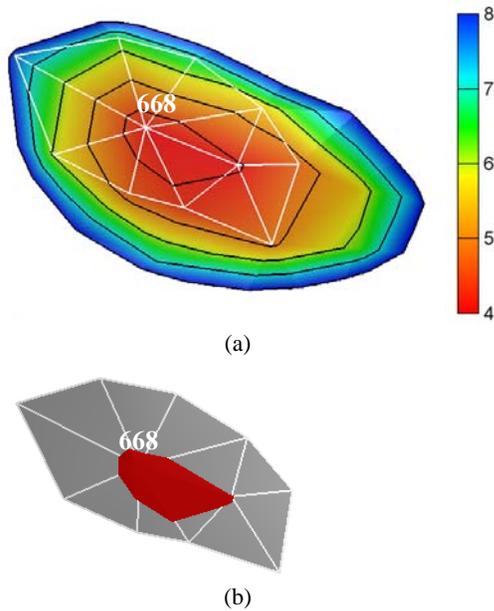


Figure 1. a) Patch of the triangulated heart surface around the focus of an activation sequence; started at node 668 at 3.38 ms; activation times in ms. b) Red area: source strength active in the gold standard method at 4 ms.

$$\text{with } g_1(\vec{x}) + g_2(\vec{x}) + g_3(\vec{x}) = 1.$$

The functions  $S(t)$  are stylized versions of the local TMP (Fig.2), shifted and scaled such to have a zero value up to the moment of local depolarization, and a unit value of its upstroke. This fast upstroke gives rise to the near-discontinuous shape of the resulting computed potentials, as is explained by using Figs. 1 and 3.

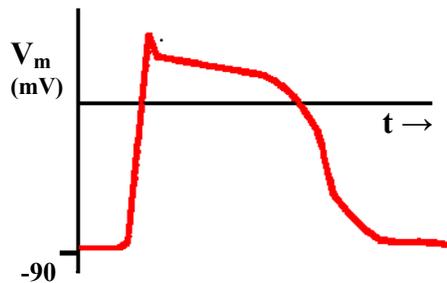
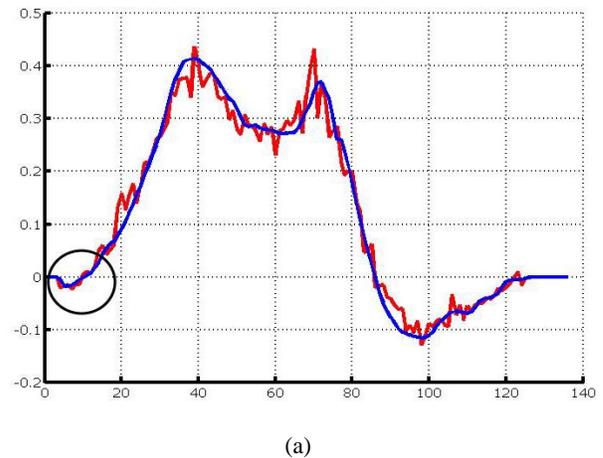
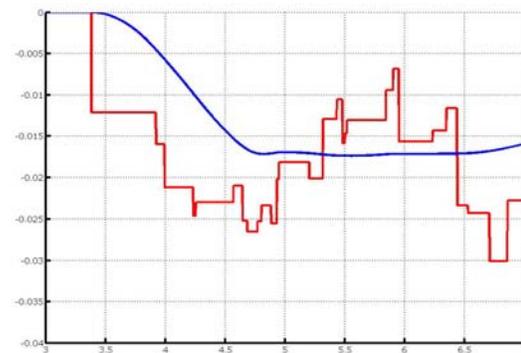


Figure 2. Stylized transmembrane potential.

Figure 1a shows a patch of a triangulated myocardial surface around the focus of an activation sequence. The activation times are color coded. A focus at node 668 at activates the local region at  $t = 3.38$  ms. As long as the subsequent activation wave has not reached any of the other nodes of the triangles sharing the focus, the contribution to the potential computed according to (3) is



(a)



(b)

Figure 3. a) The red trace shows the QRS complex computed for the activation sequence of figure 1 using the traditional method, the blue trace shows the result for the gold standard method.

b) Magnified circled part of figure 3a for highlighting discontinuities. These discontinuities are everywhere over time.

zero. Following their local activation, their contributions are added to the computed field.

Figure 3 shows (in red) the potential computed at 1 ms intervals following an activation sequence over  $S_m$  based on (3). Note the jerky nature of trace. In our previous work, these discontinuities were overcome by temporal filtering is as explained in the introduction.

In order to prevent these discontinuities without temporal filtering we developed a new handling in which, at each time sample, the contributions of sources on the triangles are restricted to the sub-triangles specifying their activated region. The blue line in Fig.3 shows the result for this new handling, which we term the gold standard method. At each time step, the gold standard method requires an updating of the geometry of the activated area and computing the corresponding contribution to the potential field, which is computationally demanding.

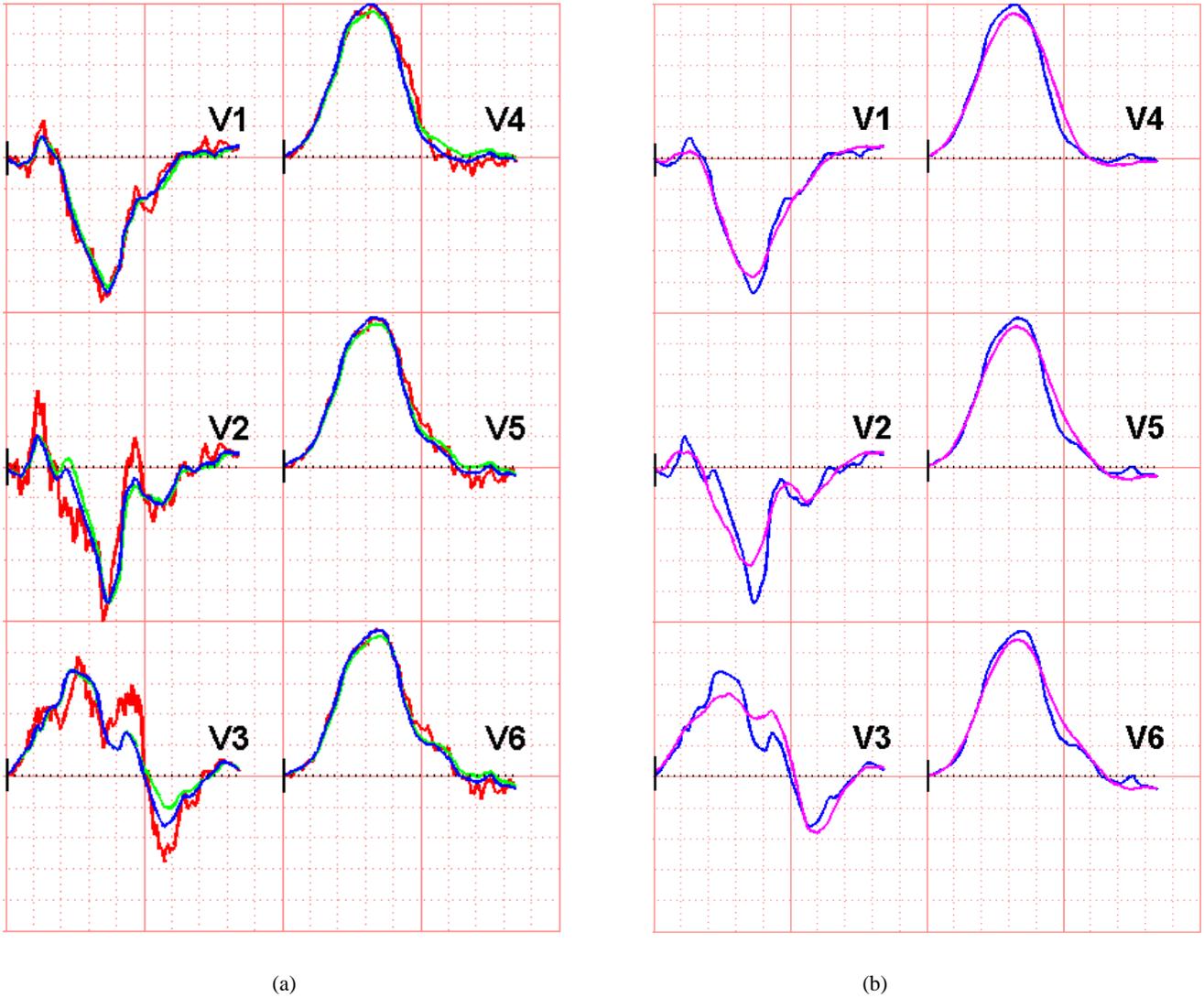


Figure 4. QRS part of reconstructed standard BSPs. a) red traces show the traditional method, green traces the fast method and blue traces the gold standard method. b) blue traces show the gold standard and magenta traces shows the filtered method used previously.

Therefore an alternative method was developed. In this method, referred to as the Fast method, the contribution based on source strength specified at a vertex of a source triangle at a particular time is weighted with the activated fraction of the triangle. For a source triangle with timing parameters  $\tau_1, \tau_2, \tau_3$  at its nodes such that  $\tau_1 < \tau_2 < \tau_3$ , the weight  $w(t)$  for that triangle is computed as:

$$w(t) = g(\tau_1, \tau_2, t) \cdot g(\tau_1, \tau_3, t) , \quad (5)$$

with, for  $i = 2, 3$ ,

$$g(\tau_1, \tau_i, t) = \frac{(t-\tau_1)}{(\tau_i-\tau_1)} \text{ if } \tau_1 < t < \tau_i$$

$$\begin{aligned} g(\tau_1, \tau_i, t) &= 0 & \text{if } t < \tau_1 \\ g(\tau_1, \tau_i, t) &= 1 & \text{if } t > \tau_i \end{aligned}$$

In the fast method the contribution of source strength of a source element increases proportionally to the fraction of the source element that has been activated. The total weight of the values at node  $j$  is the sum of the weights of all triangles of which node  $j$  is a vertex.

The resulting body surface potential,  $V_i(t)$  of the forward formulation can be written as:

$$V_i(t) = \sum_j \tilde{a}_{ij} w_{j,k}(t) S_j(t), \quad (6)$$

with  $w_{j,k}$  the weight of the contribution of the source strength specified at node  $j$  of triangle  $k$ . Note that

$$w_{j,k}=0 \text{ if } t \leq \min(\tau_1, \tau_2, \tau_3) \text{ and} \\ w_{j,k}=1 \text{ if } t \geq \max(\tau_1, \tau_2, \tau_3).$$

### 3. Results

To compare the results of the traditional and both new methods a set of depolarization times, estimated by inverse computations based on a patient's BSPs, were used. The patient data used in this paper were acquired in an earlier study which was approved by the competent Ethics Committee and which was conducted in accordance with the ethical standards established in the Declaration of Helsinki of 1946. Informed consent was obtained from all participants before enrolment in the study.

BSPs were calculated based on each of the three expressions discussed for source specifications of the EDL. In Fig.4a the resulting standard 12-lead ECGs are shown for the traditional method without temporal filtering (red), the gold standard method (blue) and the fast method (green). Figure 4b shows the waveforms of the standard 12-leads for the traditional method without filtering (red), the gold standard method (blue) as well as for the 20 ms moving average filtered traditional method (magenta).

Both the gold standard method and the fast method were able to prevent the discontinuities that are generated by the traditional method without the need for filtering. The relative difference between the standard 12-leads of the fast methods with reference to the gold standard was 9% for the sinusbeat, for the ectopic beat it was 12%.

In addition, the methods were tested on the ECG signals recorded signals sinus rhythm on a patient with complex depolarization morphology due an old myocardial infarction, which results in jerky amplitudes on its QRS complexes. In contrast to the traditional low-pass filtered signals, the new methods removed the discontinuities while not concealing the physiological short-term fluctuations in the signals (Fig.4b).

### 4. Discussion

The results of this study clearly show the elimination of the discontinuities. The physiological more realistic gold standard prevents the discontinuities that are found in the traditional method, without the requirement of temporal filtering. Moreover, it is shown that the fast method provides a fair approximation of the gold standard method based source treatment that is fast enough in interactive forward simulations of the ECG [8]

and useful in the exhaustive search on an initial estimate for the inverse solution in activation time imaging [9].

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### References

- [1] Huiskamp GJM. Noninvasive determination of human ventricular activation. Ph.D thesis, University of Nijmegen, 1989.
- [2] Tilg B, Fisher G, Wach P. Noninvasive myocardial activation time imaging: a novel inverse algorithm applied to clinical ECG mapping data. *IEEE Trans Biomed Eng* 2002; 49:1153-1161.
- [3] Huiskamp GJM, van Oosterom A. The depolarization sequence of the human heart surface computed from measured body surface potentials. *IEEE Trans Biomed Eng* 1988; BME-35:1047-1058.
- [4] Cuppen JJM, van Oosterom A. Model studies with the inversely calculated isochrones of ventricular depolarization. *IEEE Trans Biomed Eng* 1984; BME-31:652-659
- [5] Geselowitz DB. Description of cardiac sources in anisotropic cardiac muscle. Application of bidomain model. *J Electrocardiol* 1992; 25 Sup:65-67.
- [6] van Oosterom A. A genesis of the T wave as based on an equivalent surface source model. *Journal of Electrocardiography* 2001, 34s: 217-227.
- [7] de Munck JC. A linear discretization of the volume conductor boundary integral equation using analytically integrated elements. *IEEE Trans Biomed Eng* 1992; BME-39:986-990.
- [8] van Oosterom A, Oostendorp TF. The Electrocardiogram according to ECGSIM. In: Multimodal Cardiac Imaging; Olle Pahlm and Galen Wagner (eds), McGrawHill Medical NewYork, 2011; Chapter 15.
- [9] van Dam PM, Oostendorp TF, Linnenbank AC, van Oosterom A. Non-invasive Imaging of cardiac activation and recovery. *Ann Biomed Eng* 2009;37:1739-1756.

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