Quantitative Spectral Criteria for Cardiac Navigation Sampling Rate using Manifold Harmonics Analysis

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Abstract

The spatiotemporal sampling of electrograms (EGM) yielding 3-D maps of cardiac features (activation, organization, amplitude) in Cardiac Navigation System (CNS) for arrhythmia ablation, is currently driven by heuristic considerations. Manifold Harmonic Analysis (MHA) allows spectral analysis of multidimensional structures in computer graphics, however, its connection to Sampling Theory has not been fully established. Our objective was to develop a systematic and quantitative procedure to support the spatial sampling of intracardiac EGMs during electrophysiological (EPS) procedures. We used MHA for establishing quantitative spectral criteria for 3-D and 4-D maps, in connection with conventional sampling theory. After automatic segmentation in 7 Computerized Tomography images of the left atria, a 3D mesh was generated for its inner face. MHA eigendirections and spectral coefficients were calculated, showing a clear low-pass structure. Reconstruction quality was compared with the reconstruction error using mesh simplification method (Qlim) in terms of decay constant τ. Consistent τ were found when using the geometrical deviation (211.2 ± 89 points in MHA, 173.8 ± 19.7 points in Qlim, ns), but not when using mean squared error (70.5 ± 17 vs 392.6 ± 214.1, $p < 0.001$). EPS feature maps were simulated as smooth focal variations in a surrogated 4th dimension on the real atria grids, and showed a quantifiable trend to increase with the mechanism size (271-531 points for small, 135-413 for middle, 18-163 for wide size, for 85%-90% of total spectral power). MHA can be used to support the feature and spatial sampling rate in CNS-based EPS studies in connection with well-known sampling theory concepts.

1. Introduction

Sequential Cardiac Navigation Systems (CNS) are widely used in cardiac electrophysiology for visualization of informative features in a heart chamber under study (atria or ventricles), by successively sampling the intracardiac electrograms (EGM) in a number of locations with known spatial coordinates [1]. Catheter recorded EGM in sustained arrhythmias are subsequently processed in order to generate feature maps, which show either the spatial distribution of EGM activation time, or the maximum EGM voltage amplitudes, highlighting the regions with normal conduction and with scars. Also, in atrial fibrillation patients, features such as dominant frequency and regularity indices are obtained. These spatial maps support the cardiologist for determining the ablation targets [2]. CNS have even been recently proposed as a likely substitution for radioscopy in ablation or in device implantation procedures, and recent systems incorporate additional anatomical information of cardiac geometry from computer tomography (CT) or magnetic resonance images.

However, there is no clear indication about the number of spatial locations that are necessary for yielding accurate enough maps for the usual cardiac features. Given that the number or sampled EGM is a major determinant of the duration of the clinical ablation procedure, our aim was to propose a systematic and quantitative method suitable for analyzing the spatial maps quality in terms of the spatial sampling rate.

While sampling Theory gives a principled approach for making a quality reconstruction of continuous time signal from their sampled discrete-time counterparts, the generalization of Sampling Theory to high dimensional space turns to be a hard, theoretically advanced topic. Recently [3], a significative contribution has been made from computer graphics and visualization technology, in terms of an explicit method to compute a generalization of the Fourier Transform on a mesh. However, and to our knowledge, this so-called Manifold Harmonic Analysis (MHA) has not been previously used for providing spectral principles in connection to conventional sampling theory, specifically in the CNS environment.

The paper is structured as follows. In the next section, we summarize the main principles of MHA for meshes. In Section 3, we describe the image data set used in this study (atrial medical images with anatomical structure), and the proposed method to analyze the spectral content and its connection with quality geometry and EPS maps. Conclusions are finally summarized.

2. MHA spectral sampling rate

MHA Fundamentals. Fourier analysis is the classic tool for spectral analysis of unidimensional time signals and bidimensional images. The Fourier transform of a signal gives its decomposition into a linear combination of the eigenvectors of the Laplacian operator [4]. The well-known Laplace-Beltrami operator naturally extends the Fourier analysis to discrete triangle meshes, and the eigenfunctions of the Laplace-Beltrami operator, so-called manifold harmonics, correspond to the basis function [5]. In [3], the Laplacian operator, named Laplace-De Rham operator, was formulated from Discrete Exterior Calculus. This operator on 0-forms is defined as $\Delta = -\star_0^{-1} d_1^T \star_1 d_0$, where $\star$ is the Hodge star, d is the exterior derivative, and the coefficients of matrix Δ are given by

$$\Delta_{ij} = -\frac{cotan(\beta_{ij} + cotan(\alpha_{ij})}{\sqrt{|v_i||v_j|}}; \Delta_{ii} = -\sum_j \Delta_{ij} \quad (1)$$

where v_i is a vertex with coordinates (x, y, z); $|v_i|$ is the dual cell of vertex i; and angles β and α are the opposite to the edge between i and j.

The eigenvectors of Δ are orthonormal basis $H^k \in \Re^n$, named Manifold Harmonic Basis (MHB). MHB are similar to the sinusoidal basis in Fourier Transform. The spectral coefficients can be readily calculated by projecting the vertices onto the basis function H_k: $a_k = \sum_{i=1}^n v_i H_i^k$. Lower (higher) frequencies correspond to the lower (higher) eigenvalues, and give the general shape (details) of the mesh, which can in turn be reconstructed back by using the inverse of MHT, i.e., $v_i = \sum_{k=1}^m a_k H_j^k$, where m is the number of coefficients used in the reconstruction. The interested reader is referred to [3] for deep theoretical details.

Mesh Simplification Method. In our approach, we used a mesh simplification method in order to make the mesh simpler and lighter in memory, so-called *Surface Simplification Using Quadric Error Metrics* (*Qlim*) [6]. This method simplifies mesh surfaces, including color and textures, by using iterative contraction of vertex pairs or edge contraction, i.e., two vertices are merged into one, $(v_i, v_j) \rightarrow v$. The contraction is performed by minimizing the change in the appearance of the mesh by using quadratic form $\Delta(v) = v^T Q v$, where $Q = Q_i + Q_j$. Q_i (and Q_j) are the sum of the squared distance of the vertex i from the set of planes associated, and v_i is each vertex as the solution of the intersection of a set of planes. Since $Q(v)$ is quadratic, new vertex v is obtained by finding its minimum from partial derivatives $\bigtriangledown Q(v) = 0$. The algorithm can be easily extended to $\Re^n$, and in this case, the first three components of v are given by its spatial coordinates and the remaining components are given by the property values.

Procedure for MHA Based Sampling Rate Analysis. We propose a simple and principled method to analyze and quantify the spatial and feature sampling rate by using the previously described MHA. We first study the sampling rate only for the geometry, as this information is readily available in medical images, and then we extend the analysis into cardiac electric feature maps, which are naturally supported in the manifold given by the geometry. For having a gold-standard available suitable for accurate performance evaluation, we benchmarked this implementation in simulated feature maps in CNS with different degrees of surrogated arrhythmia complexity.

The effect of the spatial sampling rate for the geometry, which can be used as a mesh equivalent for the Nyquist limit, can be analyzed as follows: (1) automatic segmentation of the atrium from images of CT is performed, and a mesh of the intracardiac cavity is obtained from the segmented image; (2) the geometry of a given mesh is first transformed into a frequency space by computing its MHT, and then it is simplified by using the *Qlim* algorithm; (3) the number of coefficients in MHT is cross-checked with the number of points in the simplified mesh, in order to compare the minimum sampling rate to reconstruct the geometry of mesh; (4) the mesh is reconstructed by using MHT and *Qlim*.

Similarly, the electrophysiological feature in the maps can be transformed into a frequency space by computing its MHT, and the number of coefficients can be analyzed in order to study the effect sampling rate in its reconstruction.

3. Results

We analyzed effect of the the spatial sampling rate on the mesh reconstruction quality by means of MHA considerations, both for the application to cardiac chamber geometry from medical images, and for surrogate maps of electrophysiological features accounting for different characteristics in arrhythmia mechanisms. First, an automatic image segmentation method was used to obtain the mesh of the left atria in our database. Then, the MHT for the geometry was computed to analyze the effect of spatial sampling rate and its comparison with the number of spatial eigendirections and their spectral coefficients, and subsequently, the spectral structure (and implicit bandwidht) was checked versus the reconstruction with mesh simplification *Qlim* algorithm. Finally, the spectral dependence the electrophysiological feature in the surrogate maps with

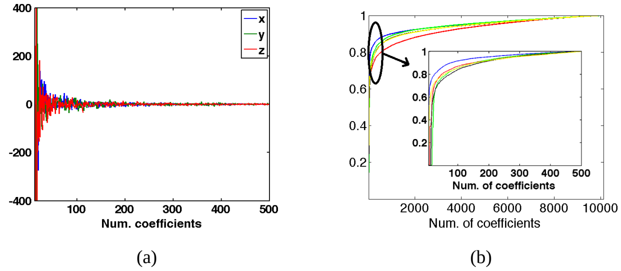

Figure 1. Spectral coefficients for the mesh geometry in an example atria (a) and the normalized accumulated power of spectral coefficients for all the atria database.

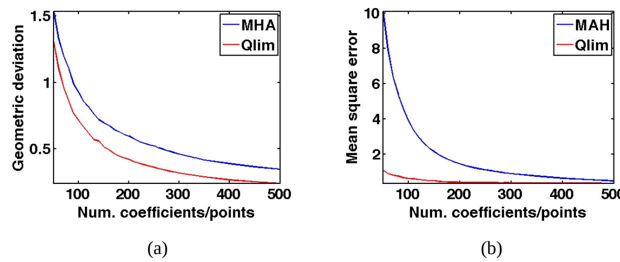

Figure 2. Mean GD (a) and MSE (b) in the reconstruction of the 7 atria with MHA and *Qlim*.

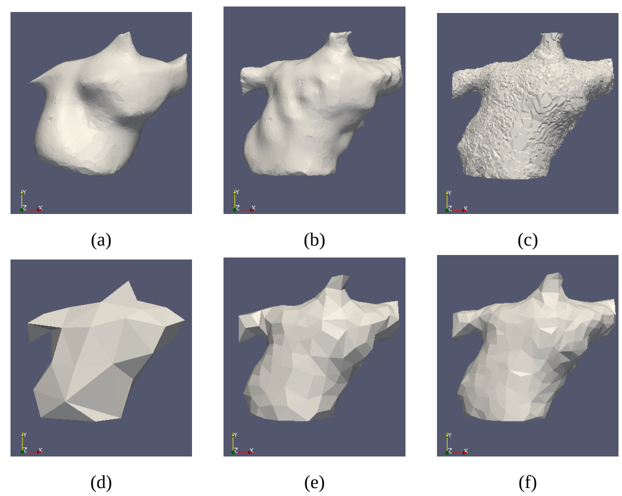

Figure 3. Atrium with 50, 250, and 500 MHA components (a,b,c), and *Qlim* simplified atrium (d,e,f).

respect to the arrhythmia mechanism was also quantified using MHT.

Geometry Segmentation. An automatic segmentation of the left atrial endocardium in CT images was made by using a region growing method, which was applied to 7 atrial images acquired with a Multi-Slice Computerized Tomography (MSCT) scanner (General Electric Healthcare LightSpeed VCT 64-slice Scanner). The algorithm consists of 4 steps: (a) the region of interest (left atrium) is selected in a representative slice of the image; (b) an anisotropic filter is applied to remove the noise introduced in the image by MSCT; (c) the image is segmented by using a conventional region growing method; (d) the mesh of the segmented volume is finally obtained. Given that meshes are composed by about hundred thousand vertices and faces, the computational load of MHT in them would be extremely high, hence, the number of vertices and faces was immediately reduced to 10,000 vertices by using *Qlim*.

MHA of Atrial Geometry. Meshes of each atria were transformed into a spatial frequency space by computing their MHT. Figure 1 (a) shows the coefficients of coordinates x, y, and z, in an example atrium. Coefficients from 500 to 10,000 were not represented because of their extremely low power. The mesh spectrum shows that the higher power of coefficients are concentrated in the lower frequencies. Spectral coefficients and their relationship to the reconstruction quality were further analyzed. Figure 1 (b) shows the normalized accumulated power of spectral coefficients, and Table 1 shows the minimum number of coefficients to obtain 85% and 90% of power. Both analysis suggest that about 250 coefficients can be an adequate number of coefficients to reconstruct the mesh geometry in this case.

Quality of the Geometry Reconstruction. We compared the number of spectral components used in the MHA reconstruction with using a limited number of mesh points (in terms of the simplified mesh given by *Qlim* simplification algorithm). Quality measurements were computed for an increasing number of coefficients and points, in terms of two merit figures, namely, the mean square error (MSE) and the geometric distance (GD), the later computing the distances between point p_i on surface S_i and the nearest point on surface S_j [7], this is, $d_g(p_i, S_j) = min_{p_j \in S_j} d(p_i, p_j)$. Figure 2 shows GD and MSE for different number of coefficients and points, and Figure 3 shows the atrium reconstructed with MHT and *Qlim*.

In order to compare the evolution of quality in MHT vs *Qlim*, GD and MSE were approximated by an exponential with decay constant τ. In GD, consistent τ was found with 211.2 ± 89 points (mean $\pm$ deviation) for MHT, and 173.8 ± 19.7 points for *Qlim* (not significant), whereas in MSE we obtained $\tau = 70.5 \pm 17$ points for MHT and $\tau = 392.6 \pm 214.1$ points for *Qlim* ($p < 0.001$). Hence, GD is a better suited quality measurement for our application, as far as GD operates with a local distance to the mesh, whereas MSE operates with the distance to the nearest point, and GD for the spectral coefficients has a close relationship with the mesh reconstruction quality.

Simulated Manifold Fields. The EPS feature maps were simulated as generated by smooth focal activations. Three different mechanism sizes (small, medium and wide) were considered, and MHT was computed after in-

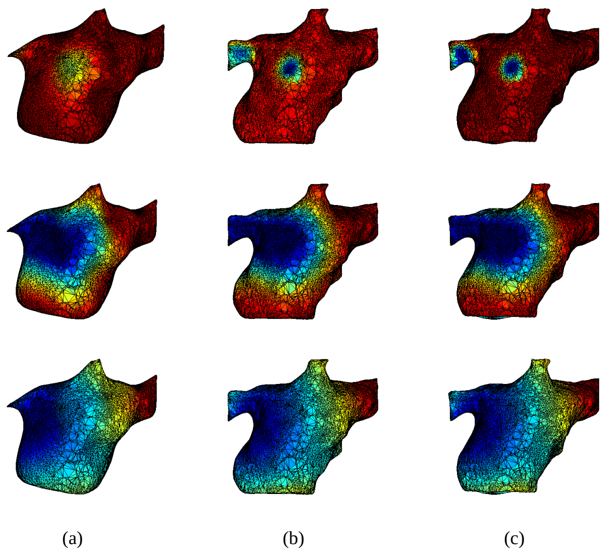

Figure 4. Two foci features in a recovered atrium with 50, 250, and 500 MHA components (a,b,c). From top to bottom: small, medium and big focus.

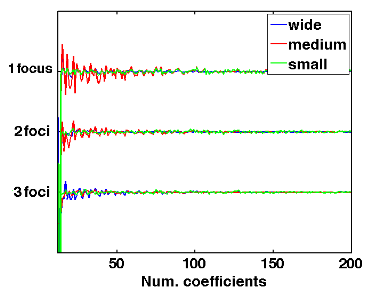

Figure 5. Spectral coefficients for the surrogated features.

cluding the feature value in the forth component of each vertex (see Figure 4). Figure 5 shows the feature coefficients for different sizes and number of foci, and Table 1 shows the minimum number of coefficients to obtain 85% and 90% of feature power in the reconstruction. Similarly to the geometry, the power spectrum of the surrogate EPS feature was concentrated in the first coefficients. The number of coefficients increased with the mechanism size, this is, smoother variations need a high number of coefficients, and then, a higher sampling rate. The number of coefficients was very different when considering 85% vs 90% of power, and also was very different in terms of the arrhythmia mechanism.

4. Conclusions

MHA has been used for giving a quantitative spectral description of the effect of sampling rate on the quality of 3D anatomical structure and of 4D cardiac feature maps

Num. coefficients for 85-90% power			
Geometry	250-960		
Features	**small**	**medium**	**wide**
1 focus	217-531	135-413	18-163
2 foci	104-238	39-188	17-90
3 foci	67-163	17-76	25-95

Table 1. Number of coefficients to recover the geometry and features atrium.

generated from catheter-recorded EGMs during EPS. This procedure can be used to support the feature and spatial sampling rate in CNS-based EPS studies. MHA of the geometry suggest that an adequate number of sampling points can be comparable to the usually achieved in current EPS procedures, and MHA of EPS feature maps shows that the sampling rate has a strong dependence on the arrhythmia mechanism. Also, the spectral profile of both geometry and feature maps has a marked low-pass character, but the quality contribution of each MHA component (and very closely, of each sampled point for reconstruction), has a very smooth decay rate. The quantitative criteria proposed in this work are currently being analyzed to provide with a clearer cut-off criteria which can be operative for EPS routine use.

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