A novel Index based on Fractional Calculus to Assess the Dynamics of Heart Rate Variability: Changes due to Chi or Yoga Meditations

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Abstract

The study aims to present and interpret a novel index based on fractional calculus (α_c) and to assess differences in RR time series dynamics in subjects during meditation. Fractional differintegration (FDI) of a RR time series provides a new time series. α_c is defined as the order of the FDI operator that provides the time series with minimum variance. Analyzed time series were obtained from the Exaggerated Heart Rate Oscillations During Two Meditation Techniques database. This database contains RR time series before and after Chi Meditation and Kundalini Yoga. It also contains recordings of elite athletes and healthy subjects while sleeping and during metronomic breathing at 0.25 Hz. α_c is significantly higher during Chi or Yoga meditation when compared with the recordings before the meditation starts or in subjects while sleeping or during metronomic breathing.

1. Introduction

Heart Rate Variability (HRV) analysis aims to characterize the variation of the time between consecutive heartbeats or RR interval. HRV is a reliable reflection of many physiological factors modulating the normal rhythm of the heart [1]. The techniques to analyze HRV can be grouped in three major (and non-independent) classes: time-domain analysis, frequency-domain analysis (including time-frequency analysis) and non-linear dynamics analysis. Time-domain analysis was the first technique to be introduced and because of its easy interpretation, the most popular tool of HRV in clinical practice. For short time recordings, two of the most employed indices are the standard deviation of the RR time series (SDNN) and that of the differentiated RR time series (RMSSD). While SDNN is regarded as an index of overall variability, RMSSD is used as a surrogate measure of vagal activity but both indices work in the same way: the computation of the standard deviation of time series that are related, in this case, by a differentiation operation.

Fractional calculus deals with the generalization of differentiation and integration to non-integer orders. The subject has gained importance during the last three decades due to its applications in various fields of science and engineering [2] such as fluid flow dynamics, electrical networks and probability but the issue is not new at all: In 1695, Leibniz and l'Hôpital enrolled in a discussion on what is the meaning of the derivative of order 0.5 instead of the classical derivative of order 1.

On the other hand, since 1980 but especially from 1990 to now, several works have focused on the fractal-like (or multifractal-like) nature of HRV time series [3] and the modelling of RR time series as time series with long-range correlations with a characteristic scaling exponent. Two of the most used processes that show long-range correlations are the fractional Gaussian noise (fGn) and the fractional Brownian motion (fBm). Both processes can be easily obtained from a white Gaussian process after proper fractional differintegration [4]. Hence, it is surprising the lack of indexes for HRV based on fractional differintegration.

The aim of this work is to present and interpret a novel index based on fractional calculus (α_c) and to assess differences in RR time series dynamics in subjects during meditation

2. Materials and methods

2.1. Fractional calculus and index proposal

For discrete time applications the Grünwald-Letnikov definition is the most appropriate to implement the fractional differintegration operator. For a certain order (α) the fractional differintegration is defined as:

$$D^{\alpha}RR(k) \cong \sum_{j=0}^{k}(-1)^{j}\cdot\frac{\Gamma(\alpha+1)}{\Gamma(j+1)\cdot\Gamma(\alpha-j+1)}\cdot RR(k-j) \tag{1}$$

being Γ the gamma function. This operator can be implemented as an IIR filter [5]:

$$D^{\alpha}RR(k) \cong \sum_{j=0}^{k} c_j^{\alpha} \cdot RR(k-j) \tag{2}$$

where the coefficients are recursively computed as:

$$c_0^{\alpha} = 1, \qquad c_j^{\alpha} = \left(1 - \frac{1+\alpha}{j}\right) \cdot c_{j-1}^{\alpha} \tag{3}$$

The order is positive for fractional differentiation and negative for fractional integration. In this work, all the fractional differintegrations have been performed using expression (2) by previously removing the mean of the RR time series for stability purposes. The application of the operator for a certain α creates a new time series that can be analyzed with any of the methods used in HRV analysis.

Our observations in actual RR time series reveal that one interesting way to exploit the differintegration operator in HRV analysis is to observe how the standard deviation of the fractional differintegrated time series change with α. Figure 1 shows this evolution for an actual RR time series. As seen, the standard deviation presents a minimum for a certain order. We define *SDFDINN(α)* as the standard deviation of the fractional differintegrated RR time series for α order. In particular: SDNN is *SDFDINN*(0) and RMSDD is *SDFDINN*(1). We also define $SDFDINN_{min}$ as the minimum standard deviation of the FDIRR set and α_c the order of the fractional differintegration that provide this minimum. We propose α_c as the index to characterize the dynamics of the RR time series. [4] describes how fractional Gaussian noise (fGn) and fractional Brownian noise (fBn) is obtained by fractional differintegrating Gaussian white noise. If a realization of a white Gaussian process is fractional differintegrated for order $-\alpha$ then the obtained time series has a scaling (or Hurst) exponent equal to α+0.5. Conversely, if a time series behaves as fGn with a scaling exponent *H:*

$$H = \alpha_c + 0.5 \tag{4}$$

because by fractional differintegrating the time series with an order α_c, a white Gaussian process will be obtained. So α_c can be regarded as an indicator of the scaling properties of the time series if it can be modeled as a monofractal time series.

2.2. RR time series in meditation, sleep and metronomic breathing

In order to assess if α_c has the ability to distinguish among different physiological states in healthy subjects, we have employed the "Exaggerated Heart Rate Oscillations During Two Meditation Techniques" database (EHRO database) [6] that is available at www.physionet.org. The database contains 5 different groups of healthy subjects. The first group (CH group) corresponds to eight subjects where the RR time series was measured before and during Chi Meditation. Each series has duration of about one hour. The second group (Y group) consists of four subjects and the RR time series were obtained before and during Kundalini Yoga meditation. Time series durations range from 17 to 47 minutes. The third group (SL group) has RR time series with durations of around 6 hours each from eleven volunteers recorded while sleeping. The fourth group (I group) consists of nine elite athletes measured during sleep. The duration of each RR time series ranges between one and two hours. Finally, the fifth group (PB group) consists of fourteen volunteers measured while breathing at 0.25 Hz for 10 minutes.

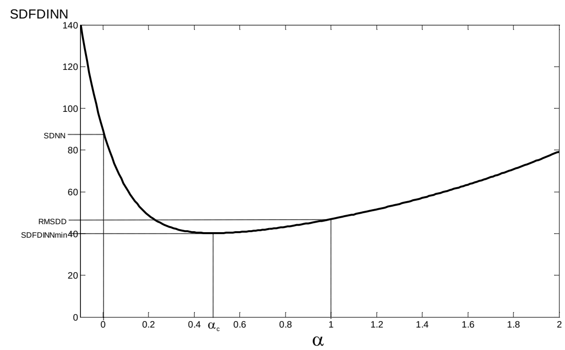

Figure 1. Evolution of the standard deviation of the fractional differintegration of an actual RR time series with the order of the operator. SDNN and RMSDD are particular values in this evolution while the minimum standard deviation provides the characteristic order α_c.

2.3. Signal processing and statistical analysis

Because the RR time series in CH, SL and I groups are longer than in the other groups, for each time series in these groups we have extracted a time series with 1000 beats. The criterion to select the time series has been the maximization of the stability of the mean of the RR time series using the Lagrange Multiplier statistic (LM) described in [7]. For each time series, the LM statistic has been computed in a sliding window of 1000 consecutive RR time samples using a displacement of only one beat. The time subseries with the lowest statistic has been chosen due to its stability in the mean heart rate. Figure 2 shows a RR time series of the CH group as well as the LM statistic and the selected time subseries.

For each time series, α_c has been estimated by minimum search using the golden section considering orders between -3 and 3. The minimization parameter is the standard deviation of the time series obtained by applying equation (2) to the original time series. The recursive search algorithm stops when the absolute difference in the output with the previous iteration is lower than 0.001

Mean and standard deviation of α_c has been obtained for each group and in groups CH and Y for the time series before and during meditation. Paired t-tests were employed to assess if there are significant differences associated to the meditation task respect to the basal state. Moreover, the results for the time series among groups were tested using t-tests.

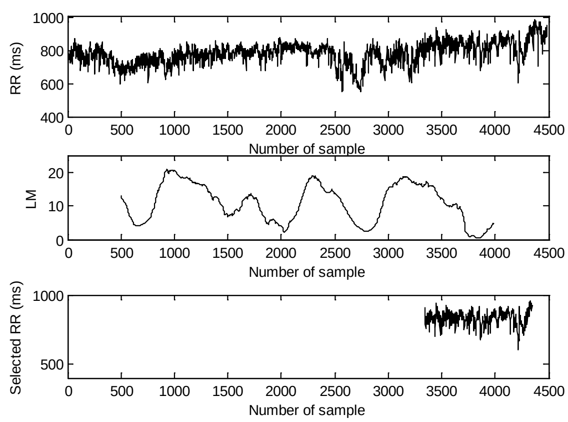

Figure 2. An example of selection of a time subseries with maximum stability of the mean. The whole time series, the evolution of the LM statistic and the final chosen time series are shown.

Table 1. Results of proposed index for the different groups of the database.

Group	N	State	Mean	Standard deviation
CH	8	Basal	0.71	0.19
		Meditation	1.14	0.30
Y	4	Basal	0.66	0.16
		Meditation	1.32	0.25
SL	11	Sleeping	0.58	0.39
I	9	Athletes sleeping	0.44	0.22
PB	14	Metronomic breathing	0.47	0.26

3. Results and discussion

Table 1 shows the results of α_c for the different groups. During Chi meditation, α_c is very significantly higher ($p<0.001$) than in the basal state. While performing a Kundalini Yoga meditation, α_c is also significantly higher than in the basal state ($p=0.012$). In fact, for all the tested subjects, α_c is always higher during meditation than in the basal recording. t-tests show no significant differences between the basal states of groups CH and Y nor between the meditation states. The index in the basal state of CH group is significantly higher than in the I ($p=0.019$) or PB ($p=0.028$) groups while there are no significant differences with the SL group. During Chi meditation, α_c is very significantly higher than in I and PB groups ($p<0.001$) and significantly higher ($p=0.003$) than in the SL group. The same significances are obtained when comparing the Kundalini Yoga meditation and the SL, I and PB groups.

Results show that α_c is low in relaxed states (while sleeping, in supine position breathing periodically) but high while meditating. Figure 3 shows the whole results grouped by state. Unfortunately, the number of measured subjects is quite low (especially for the Kundalini Yoga meditation) so the results must be considered as very preliminary.

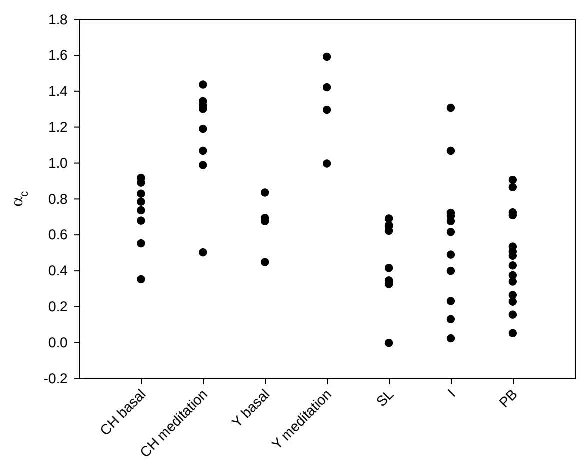

Figure 3. Complete set of results for α_c grouped by state.

Meditation is regarded as a tool to reduce stress. At first glance, results in the index would be closer to relaxed states (as sleeping certainly is) than to the basal measurement. Nevertheless, results show that α_c increase (instead of decreasing) during meditation. During the studied meditation techniques, the breathing rate is very low and this pseudoperiodic component can be the cause of the high measured α_c. Other kinds of meditation techniques not based on controlling the breathing should be studied to ascertain if the rise in the index is due to the slow breathing or there are other modulating sources that affect the dynamical properties of the RR time series.

4. Conclusions

An index (α_c) based on fractional differintegration is proposed to assess changes in the dynamics of RR time series. α_c is significantly higher during Chi or Yoga meditation when compared with the recordings before the meditation starts or in subjects while sleeping or during metronomic breathing.

Acknowledgements

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