

Avoiding the Inverse Crime in the Inverse Problem of Electrocardiography: Estimating the Shape and Location of Cardiac Ischemia

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Abstract

The Inverse Problem of electrocardiography (IPE) can be summarized as the characterization of the electrical behavior of the heart using measurements obtained by electrodes that are not directly in contact with the cardiac surfaces. Given a data ensemble provided by electrodes, the solution of the IPE requires the design of a mathematical procedure that matches a theoretical model of estimated measurements with that ensemble of data. Common tests of inversion procedures were often made with synthetic data using the same model for computing both predicted and estimated measurements, yielding into overoptimistic results; this is called the Inverse Crime. In practice, the test of an inversion process avoiding the Inverse Crime could be done using a model for the numerically produced simulated data and a different one to invert the data. This work shows the behavior of a procedure designed to characterize regions in the heart with a lack of blood supply (ischemia) avoiding the Inverse Crime. Realistic and experimentally supported models constitute the forward procedure (the Luo-Rudy model for the electrical activity and the volume conductor theory for simulating the electrode measurements) while a simple phenomenological model (the two-current model proposed by Mitchell and Schaeffer) is used during the inversion process.

1. Introduction

Solving inverse problems, related to some application, requires a parametric adjust of a theoretical model, usually called inverse model, in order to make as close as possible measurements obtained from the theoretical model and the real data.

The use of the same mathematical model to obtain the predicted and estimate data can give unrealistic results, which constitutes the so-called Inverse Crime (IC) [1]. In practice, in order to mitigate the IC, authors usually introduce some discrepancies in the inversion procedure, for example, adding noise to predicted data [2] or using differ-

ent meshes in the forward and the inverse procedures [3]. Other possibility, which has not been too explored in the Inverse Problem of Electrocardiography (IPE), is the use of different mathematical models for the forward and the inverse procedures.

In this work, we present an inversion procedure avoiding the IC in the framework of the IPE. Specifically, we focus in the characterization of cardiac ischemic areas (location and shape reconstruction). Previous studies have analyzed the characterization of ischemia regions from remote signals [4, 5]. However, in all these works, the mathematical model to obtain the real (forward) and the predicted (inverse) data are exactly the same, thus they are under the IC scenario.

The aim of this study is to extend the previous work [5] avoiding the IC. For this purpose, we take into account the following considerations: Firstly, we do not assume control over the initial activation so we need to estimate the initial condition using an activation detection algorithm [6]; Secondly, we consider two different mathematical models to describe the electrical behavior of the heart. The forward computation is done with a model that resembles main effect of an ischemia. On the other hand, for the inverse procedure, we choose a simpler phenomenological model in which modifying properly some of its parameter it is possible to simulate an ischemic disorder.

2. Mathematical framework

In this section, we present the mathematical models used for the forward and inverse problem.

The forward model refers to the mathematical model that realistically describes the ionic processes involved in the generation of the cardiac action potential and its propagation through the cardiac tissue [7]. On the other hand, the inverse procedure is an iterative process which requires to calculate the solution of an inverse model for cardiac tissue. This constitutes a computationally intense procedure, and therefore we selected a simple phenomenological model for the inverse procedure [8].

2.1. The forward model

In order to simulate ischemia with a high degree of electrophysiological detail, we implement a modified version of the Luo-Rudy dynamic action potential model (LRd00) [7]. This mathematical description incorporates detailed information of the ischemic electrophysiological effects, including information at ionic level. This formulation includes the ATP sensitive potassium current activated during ischemia, see reference [9]. The electrical activity of a cardiac tissue affected by regional ischemia is simulated considering the three major effects of this pathology: hyperkalemia, acidosis, and hypoxia; based on the parameters presented in [10].

2.2. The inverse model

The two-current model (TC), based on the model proposed by Mitchell and Schaeffer [8], consists of just two ordinary differential equations for two variables: the transmembrane voltage $v(t)$ and an inactivation gate variable $h(t)$. The voltage equation is defined as follows

$$\frac{dv}{dt} = J_{TC} = J_{stim}(t) + J_{in}(v, h) + J_{out}(v), \quad (1)$$

where J_{stim} represents the initial stimulus, J_{in} and J_{out} denotes the inward and outward currents, respectively:

$$J_{in}(v, h) = \frac{h(1-v)(v-v_{rest})^2}{\tau_{in}}, \quad (2)$$

$$J_{out}(v, h) = -\frac{v-v_{rest}}{\tau_{out}}. \quad (3)$$

The gating variable $h(t)$ regulates inward current flows and obeys the following equation

$$\frac{dh}{dt} = G(v, h) = \begin{cases} (1-h)/\tau_{open}, & v < v_{crit} \\ -h/\tau_{close}, & v \geq v_{crit} \end{cases} \quad (4)$$

This model contains four time constants, which correspond to the four phases of the cardiac action potential: initiation, plateau, decay and recovery. As we show in Fig. 1, healthy and ischemic conditions can be modeled modifying τ_{in} , and v_{rest} (resting potential), see [5] for more detail.

2.3. Model of cardiac tissue

Despite the discrete nature of cardiac cells structure, at the macroscopic scale cardiac tissue behaves as a functional syncytium. This permits to consider cardiac tissue (Ω) as an excitable medium in with the membrane potential

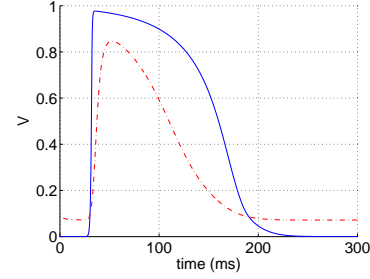


Figure 1. Resulting action potential for healthy (solid line) and ischemic cells (dashed line) simulated with TC model

propagation can be mathematically described according to the following reaction-diffusion equation

$$\frac{\partial v}{\partial t} = \nabla \cdot (D\nabla v) + J, \quad (5)$$

with the no-flux boundary condition and appropriated initial conditions. Here D represents the scalar diffusion coefficient and the reactive term, J , is taken as the ion current provided by LRd00 or TC. Equation (5) represents the so-called monodomain approximation which have been extensively used to simulate the propagation of the membrane potential through the cardiac tissue.

2.4. Model of remote recordings

In order to measured the electrical activity at the cardiac tissue, we consider in the present study remote recording measurements modeled as point electrodes. Following the volume conductor theory [11], the electric potential registered at point \mathbf{r}_i is given by

$$\varphi^i(t) = \frac{1}{4\pi\sigma} \int_{\Omega} \frac{\nabla \cdot (D\nabla v(\mathbf{r}, t))}{|\mathbf{r} - \mathbf{r}_i|} d\Omega, \quad (6)$$

where σ represents the medium conductivity (assumed homogeneous), and $v(\mathbf{r}, t)$ is solution of (5). This expression, together with the cardiac excitation model and propagation model used, comprise a complete description of the mathematical framework.

3. Inverse procedure

The proposed method is divided into two parts. Firstly, the activation detection algorithm, and secondly the shape reconstruction algorithm.

3.1. Activation detection

The activation detection sequence is estimated as follows [6]: (i) filtering the intracardiac recordings with a

bandpass of 40-250 Hz; (ii) rectification and lowpass filtering at 20 Hz. This process extracts a time-varying waveform proportional to the high-frequency components in the original intracardiac recording. The time instant of the local maximum represents an estimate of the activation time.

3.2. Shape reconstruction

Let $\varphi_{LR}^i(t)$ be the data obtained from the solution of the LRd00 model in a cardiac tissue with ischemic regions. We consider this as the objective ischemic area to be estimated. On the other hand, let $\varphi_{TC}^i(t)$ be the measurements obtained for the TC model for ischemic settings characterized by some parameters distributions $\tau_{in}(\mathbf{r})$ and $v_{rest}(\mathbf{r})$.

The aim is to estimate the shape and locations of real ischemic areas adjusting the parameter distributions of τ_{in} and v_{rest} , minimizing the cost functional defined as the misfit of $\varphi_{LR}^i(t)$ and $\varphi_{TC}^i(t)$:

$$\mathcal{J}(\tau_{in}, v_{rest}) = \frac{1}{2} \int_0^T \sum_{i=1}^N \|\varphi_{TC}^i(t) - \varphi_{LR}^i(t)\|^2 dt \quad (7)$$

where T represents the recording time and N the number of electrodes.

In our shape-based approach, we follow an iterative scheme in which the cost functional is reduced at each step of the reconstruction process. Therefore, we find a direction in the space parameter such that the cost functional decreases. The ischemic shape is described using level set techniques, see [5] for details.

4. Results

We consider a bi-dimensional cardiac tissue of size $\Omega = 5.5 \times 5.5 \text{ cm}^2$ with two different disconnected affected areas. We use an arrangement of 25 electrodes equidistantly distributed located at 0.5 cm above the tissue (see Fig. 2 (a)). In this configuration, we obtain our *real* recording ($\varphi_{LR}^i(t)$) solving the LRd00 model using a semi-implicit finite differences scheme over a 100×100 spatial grid and using $\Delta t = 2.4 \cdot 10^{-3}$ ms for temporal discretization. The stimulation protocol consist on a plane wave front pulse J_{stim} applied on the left side of cardiac tissue. For this experiment, the reconstruction algorithm is applied to a single cardiac period of length $T = 300$ ms in the steady state.

Figure 2 (b) shows the activation time map and the conduction velocity (arrows) estimated by the activation detection algorithm applied to the 25 remote recordings obtained by the LRd00 model. Note that the activation times correspond to depolarization time instants of the intracardiac recordings, which in this case are not much affected

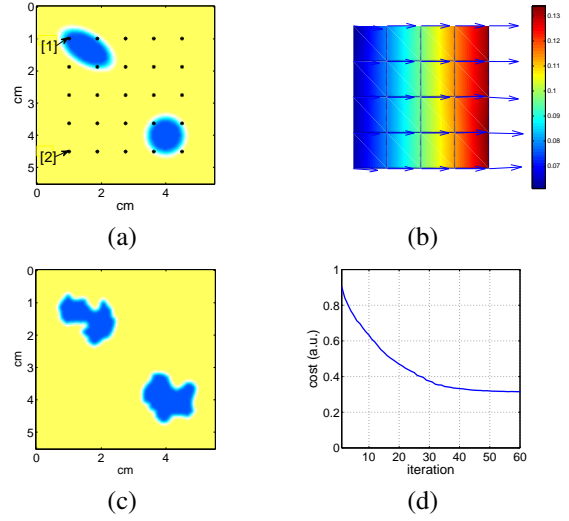


Figure 2. Numerical experiment: reconstructing two non-connected ischemias. (a) *Real* experimental setup. The positions of the electrodes are shown with small stars placed at 0.5 cm above the tissue. The ischemic areas are represented with blue color. (b) Activation times map estimation. (c) Final reconstruction profile obtained after 60 iterations. (d) Evolution of the cost functional during the reconstruction process.

by the presence of ischemia, and consequently the activation sequence resembles a plane wave propagation.

At each step of shape reconstruction algorithm we solve the TC equations using the same numerical method used for LRd00. In these case we can use a time step $\Delta t = 0.24$ ms.

The shape reconstruction algorithm starts using as initial guess a circular ischemic area located at the center of the tissue. At each iteration $\tau_{in}(\mathbf{r})$ and $v_{rest}(\mathbf{r})$ are modified reducing the cost functional (7) (as we can see in Fig. 2 (d)). The guess ischemic region breaks into two pieces moving toward the right locations. The shapes and sizes of these disconnected areas are adjusted until the stopping criterion process is fulfilled. As we show in Fig 2 (c) where the final reconstruction is represented, a good approximation of the locations, shapes and sizes is obtained after 60 iterations. At this time, the process is stabilized, presenting the cost functional a stationary behavior (see Fig. 2 (d)).

In Fig. 3 we show the recordings obtained for two different electrodes associated to the final reconstruction compared with the *real* ones. As we can see, at the end of the procedure, the morphology of both is similar to the real measures (φ_{LR}^1 and φ_{LR}^2 , respectively).

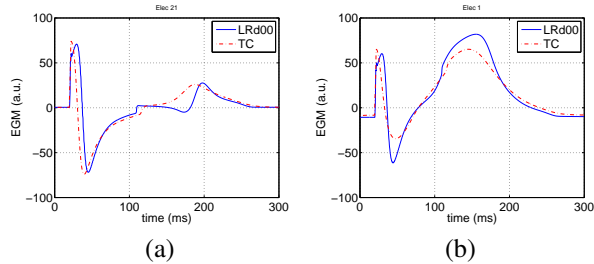


Figure 3. Non-contact recordings obtained by the electrodes marked as 1 and 2 in Fig. 2 (a). The solid line represents the real data, and the dashed line represents the recording associated to the final reconstruction.

5. Discussion

In this work we wanted to address the problematic associated to the called IC in the particular case of IPE to reconstruct an ischemic area. We have focused on issues related to the modeling of ionic currents.

We have verified that a simple model, from a computationally point of view, can be used to reconstruct non-connected ischemic areas, providing information about their location, shapes and sizes. Although the input data of the problem are real data, they are obtained by a experimentally tested model (LRd00). This is a necessary step toward a clinical application of the procedure.

We believe that the implementation of algorithms to solve the IPE may not need the use of very detailed and computationally expensive mathematical models to characterize some types of cardiac pathologies.

In the future, the presented method could be adapted to estimate, in some way, which is the degree of an ischemia.

Likewise, we expect to check this procedure in more realistic geometric situations, incorporating a 3D model.

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