Coherence as a Measure of Noise in the ECG

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Abstract

The 12-lead ECG can be described as the projection of a time-varying dipole on the lead vectors. This dipole-source assumption implies that the different leads must show a high coherence and that deviations from values close to unity may indicate the presence of noise.

We evaluated the average coherence in the band of 5 Hz to 45 Hz in 309 ECGs and compared the coherence measure with an automatic out-of-band noise evaluation, quantified at a scale from 0 (clean) to 10 (very noisy).

The generalized magnitude squared coherence (GMSC), which is a global measure of the linear relationships among the leads, was used.

A high coherence (GMSC>0.95) was always associated with very low noise (level 0 or 1) and 0.9<GMSC<0.95 was associated with levels 0-2. For noise levels > 2 the GMSC was <0.87 in all cases. There were no cases of high noise level and a high coherence, however, in 32% of the cases with noise level 0 or 1 the GMSC was <0.8.

For low-noise ECGs the GMSC is close to unity, with a good correlation between noise level and GMSC for normal ECGs. However, a low coherence also seems to be associated with abnormalities in the ECG, which will be subject for further study.

1. Introduction

One of the simplest models of the Electrocardiogram (ECG) is based on the projection of a fixed dipolar source on the lead vectors, a concept that was conceived by Einthoven *et al*. [1]. Such a model suggests that the ECG-leads are strongly coherent, i.e., that pairs of leads have magnitude squared coherences (MSC) that are close to unity.

Noise, on the other hand, with the exception of power line noise, will typically have a low coherence across the leads although pairs of leads that pick up noise from the same source may have a more coherent noise.

Generally speaking then, a noisy ECG will present with a lower coherence in the signal band than a clean ECG, whereas the coherence will generally be low outside the signal band. The MSC thus may be a simple measure of noise within the signal band of the ECG, in contrast to other noise measures, which are typically defined outside the signal band.

In the case of two signals the MSC is the magnitude squared of the complex coherence spectrum of the signals, which basically is the normalized cross-spectrum or the cross-spectrum of the pre-whitened signals. This notion was generalized by Ramírez *et al*. [2] for multiple signals as the Generalized MSC (GMSC). The GMSC of the ECG thus gives a global measure of the MSC for multiple leads instead of the pairwise MSCs.

In this study we calculated the GMSC for a variety of ECGs and we compared the average in-band GMSC with out-of-band signal (noise) levels.

2. Methods

A collection of 309 standard 12-lead ECGs, sampled at 500 Hz, with various aetiologies, were classified on a scale from 0 (virtually noiseless) to 10 (very noisy, useless for diagnostics). The method to obtain the noise levels was developed by GE Healthcare: first an upper threshold and a lower threshold was set based on the current signal value and then the number of times the signal swings from above the upper threshold to below the lower threshold was counted. The count was then normalized to a final high-frequency noise score based on a training set with clean ECG signals.

There were 9, 212, 70, 11, 5, 1, and 1 ECGs in the respective classes 0, 1, 2, 3, 4, 5, and 6.

For two signals $x = x(t)$ and $y = y(t)$ the complex coherence spectrum is defined as

$$C_{xy}(\omega) = \frac{S_{xy}(\omega)}{\sqrt{S_{xx}(\omega)S_{yy}(\omega)}},$$

where $S_{xy}(\omega)$ is the ensemble average of the cross spectra of x and y. $S_{xx}(\omega)$ and $S_{yy}(\omega)$ are the ensemble averages of the power spectra of x and y. Often the estimate of $C_{xy}(\omega)$ is implemented as the average of 50% overlap time-windowed segments of the signals (Welch's method), but here we have used spectral smoothing [3] using a Gaussian kernel, which gives similar results as compared with windowing in the time domain.

Coherence spectra were calculated for all possible pairs of leads among the eight independent leads I, II, V1-V6, yielding the 8x8 coherence matrix:

$$K(\omega) = \begin{bmatrix} 1 & C_{I,II}(\omega) & \cdots & C_{I,V6}(\omega) \\ C_{II,I}(\omega) & 1 & \cdots & C_{II,V6}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ C_{V6,I}(\omega) & C_{V6,II}(\omega) & \cdots & 1 \end{bmatrix}.$$

The GMSC can now be obtained as

$$\gamma^2(\omega) = \left[\frac{\lambda_{max}(K(\omega))}{L-1}\right]^2,$$

where $\lambda_{max}(K(\omega))$ is the maximum eigenvalue of $K(\omega)$ and L is the number of leads (in this case L=8). 1-$\gamma^2(\omega)$can be interpreted as the incoherent-noise fraction for radial frequency ω.

Subsequently the average of $\gamma^2(\omega)$ over the frequency band of f_1=5 to f_2=45 Hz was calculated numerically to estimate the in-band signal fraction:

$$\gamma^2_{f_1,f_2} = \frac{1}{2\pi(f_2 - f_1)} \int_{2\pi f_1}^{2\pi f_2} \gamma^2(\omega) d\omega.$$

3. Results

A high coherence ($\gamma^2_{5,45} > 0.95$ was always associated with very low noise (level 0 or 1) and 0.9 < $\gamma^2_{5,45}$ < 0.95 was associated with levels 0-2.

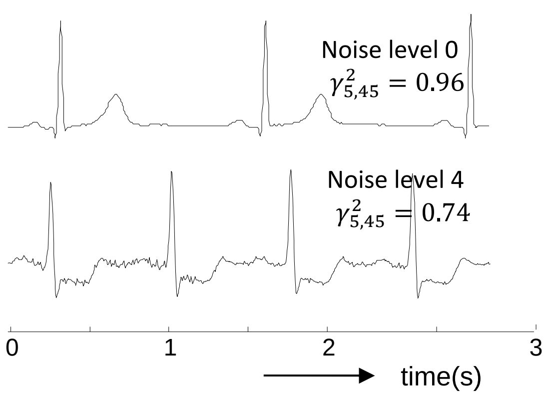

Figure 1. ECG tracings, lead V5; Upper: noise level 0, $\gamma^2_{5,45} = 0.96$; Lower: noise level 4; $\gamma^2_{5,45} = 0.74$.

For noise levels > 2 the $\gamma^2_{5,45}$ was < 0.87 in all cases. There were no cases of high noise level and a high coherence, however, in 32% of the cases with noise level 0 or 1 the $\gamma^2_{5,45}$was < 0.8.

Figure 1 shows two traces (lead V5) of two different subjects. The upper trace is from a virtually noiseless ECG, whereas the lower trace is from a moderately noisy ECG. The in-band average GMSCs are 0.96 and 0.74 respectively. Figure 2 shows the GMSCs as functions of frequency for the same subjects. The low-noise ECG has a GMSC which is close to unity over the whole signal band, whereas the ECG with noise level 4 shows a decreased GMSC for frequencies beyond 20 Hz, which agrees well with the appearance of high-frequency noise in the lower trace in figure 1.

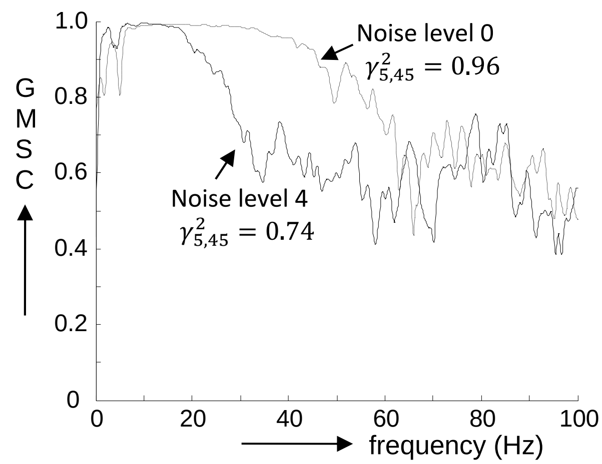

Figure 2. GMSC as a function of frequency for the ECGs indicated in Figure 1. Note the high coherence for low frequencies (up to 20 Hz) also in the signal with noise level 4.

4. Discussion

The generalized magnitude squared coherence is a simple non-parametric way to estimate the amount of noise in the ECG, with the appealing property that the actual noise within the frequency band of interest can be estimated directly.

A drawback might be that GMSC is a measure only of incoherent noise but a further analysis of the pair wise MSCs may give a better understanding of the noise distribution across the leads.

For low-noise ECGs the GMSC was close to unity in the whole signal band and in general there was a good correlation between noise level and GMSC for a variety of ECGs. It remains to be seen whether a low coherence may also be associated with abnormalities in the ECG, a topic that will be subject for further study. However, the high values for GMSC for the low-noise ECGs indicate that normally ECGs do indeed possess a high inter-lead coherence.

Noise levels out-of-band (> 60 Hz) cannot be considered as a gold-standard for the in-band (5-45 Hz) noise levels. Nevertheless, the two methods of noise evaluation were in good agreement, although further studies to merit the usefulness of GMSC have to be carried out.

5. Conclusion

GMSC is a promising method to evaluate the noise in the ECG frequency band.

Acknowledgement

This work was funded by the Danish Council for Strategic Research.

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