

# Local Regularization of Endocardial and Epicardial Surfaces for better Localization of Ectopic Beats in the Inverse Problem of ECG

Danila Potyagaylo, Walther HW Schulze, Olaf Dössel

Institute of Biomedical Engineering, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany

## Abstract

*The problem of non-invasively finding cardiac electrical sources from body surface potential maps (BSPM) is ill-posed. A standard Tikhonov regularization approach to the problem produces a solution biased toward the electrodes and thus to the left ventricular epicardium, which limits its potential to reconstruct endocardial sources. In this work we consider a transmembrane voltages based inverse problem of ECG for the identification of extrasystole origins from simulated BSPM. With use of a pair of heart wall epicardial/endocardial extrasystoles and a pair of septal ectopic foci we demonstrate the performance of the inverse procedures while firstly solving the problem for all nodes, then for epicardium and endocardium separately. Based on the observations and the logic behind the gradient of sources we define simple rules on how to classify an extrasystole under consideration according to these 3 reconstructions. Furthermore, when the amount of noise is known, we propose a new method with two regularization parameters which assign different weightings to endocardial and epicardial components of the solution.*

## 1. Introduction

The goal of ECG imaging is to non-invasively estimate electrical activity of the heart from body surface potential maps (BSPM). This field of research has been receiving growing attention due to its huge potential in cardiology. One application could be a better ablation planning and real-time guided treatment of ventricular arrhythmias. For this purpose it is necessary to reliably reconstruct electrical potentials on both epicardial and endocardial surfaces of the heart. In clinical environment, however, the ECG-imaging methods were mostly applied either to epicardial or endocardial surface [1–3] and only the latest works show the ability of solving the inverse problem of ECG for electrical potentials on both surfaces simultaneously or even for the transmural transmembrane voltages (TMV) [4, 5].

In the present work we investigate the possibility of re-

liable identification of ventricular extrasystole origins corresponding to the early activation times without applying sophisticated spatio-temporal regularization techniques for the whole QRS-complex. As shown in [6] many ventricular ectopic foci originating in different places in the heart produce indistinguishable BSPMs for a time step shortly after the abnormal heart beat initiation. Therefore when performing a standard Tikhonov regularization on these BSPMs, the optimal solution would not necessarily be the correct one. In these cases the solution is biased toward the 'outer' sources [7, 8], i.e. those situated closer to the measurement points. In the interpretation of the ECG imaging it means that the epicardial points of the left ventricle are favored over endocardial ones.

To counterbalance this situation we solve two additional problems for endocardial and epicardial nodes separately and then introduce a regularization vector parameter in order to assign different weights for epicardial and endocardial surfaces during the inverse procedure.

## 2. Methods

With the quasi-static assumption and the assumption of equal anisotropy ratios in the extracellular and intracellular spaces in the heart one can establish a linear relationship between the TMV  $x$  on the heart surface and the electrical potentials on the body surface  $y$ , defined by a lead-field matrix  $A$  [9]. As the human body acts as a passive volume conductor and smears out the signals produced by the heart, the problem of finding the sources from the measured BSPM becomes ill-posed, meaning that the inversion of the matrix  $A$  without additional a-priori information about the solution is unstable with respect to noise and model uncertainties.

A standard mathematical tool for treating ill-posedness is narrowing a space of possible solutions and translating the problem into the class of well-posed problems defined by Tikhonov [10]. This can be achieved by adding some a-priori information into the system and thus solving for a solution possessing certain properties, e.g. spatial smoothness. Then the discretized problem can be written in the

following form:

$$x = \operatorname{argmin}\{\|Ax - y\|_2^2 + \lambda\|Lx\|_2^2\} \quad (1)$$

with  $L$  matrix being a discrete approximation of e.g. the Laplace operator,  $\lambda$  is a regularization parameter responsible for the trade-off between the data misfit and a-priori information.

If we want to reconstruct only endocardial or epicardial TMV, then we can eliminate the corresponding columns in the matrices  $A$ ,  $L$  and solve a reduced optimization problem (1).

In case of a scalar parameter  $\lambda$  all the solution components are regularized equally. For a problem specific local regularization making emphasis on certain heart regions we introduce a vector regularization parameter  $\lambda = \{\lambda_1 \dots \lambda_n\}$ , where  $n$  is the number of unknowns in vector  $x$ . Then the problem (1) becomes

$$x = \operatorname{argmin}\{\|Ax - y\|_2^2 + \|\Lambda Lx\|_2^2\} \quad (2)$$

where the matrix  $\Lambda$  is diagonal with the vector  $\lambda$  on its main diagonal. In general one can assume all the components in  $\lambda$  to possibly take different values. The problem (2) would have in this case twice as many unknowns as (1) and become non-linear with respect to the joint vector  $\{x, \lambda\}$ . In this work we will consider only two possible values in the vector  $\lambda$ : one for endocardial nodes and one for epicardial nodes.

To cover the whole solution space we must consider all the possible regularization parameters combinations  $(\lambda_1, \lambda_2) = [\lambda_{min} \dots \lambda_{max}] \otimes [\lambda_{min} \dots \lambda_{max}]$ . Without any information about noise and model errors, an analog of the L-surface [11] must be analyzed, otherwise we can pick up the solution that matches to the known data misfit  $\|Ax_{true} - y\|$ .

### 3. Results

All the forward and inverse calculations were performed using a realistic human geometry containing lungs and heart as inhomogeneities. In order to demonstrate the problem of endocardial source reconstruction and to show the effects of local regularization we considered 4 simulated ventricular extrasystoles. As somewhat easier cases we examined two extrasystoles originating on the endo- and epicardial surfaces of the left ventricle, which were placed symmetrically with respect to the heart wall. Another, more difficult, pair of extrasystoles was placed in the left and right ventricular septal parts of the heart surface. All BSPM were contaminated with 20 dB Gaussian white noise. In cases where only one regularization parameter was involved we used the L-curve criterium to determine  $\lambda$  in an optimal way [12].

#### 3.1. Tikhonov reconstruction

To give the reader a scent of the problem, in Fig. 1 (first and second lines) we provide pictures of the simulated and Tikhonov reconstructed TMV for the time steps when the signal already became transmural in order to show the robustness of our approach with respect to a taken time point. From these reconstructions we learn that the best solution is achieved in case of an epicardial extrasystole, while for the endocardial foci the regularization 'pushes' the strong (in amplitude) sources to the epicardium.

#### 3.2. Endocardial imaging

In these experiments we restricted the solution to the endocardial surface, meaning that we solved a Tikhonov regularization problem only for the endocardial nodes. Neglecting the lead-field columns corresponding to the epicardial points is equivalent to setting the TMV for these nodes to 0. Now we should recollect that the BSPMs are generated by the gradient of the TMV and not the TMV itself. This property of the sources makes the solution insensitive to a shift in values. Thus when assuming the epicardial voltages to be 0 mV, the inverse procedure tries to adjust the unknowns on the endocardium in such a way that the gradient of the TMV and thus the BSPMs match the true ones.

From the reconstruction results given in Fig. 1 (in the fourth line) one can see that in 3 cases of endocardial ectopic foci the solutions obtained with this approach reproduce the simulations, while for the epicardial extrasystole we observe an inverted sign of the TMV (so that the true gradient is restored) but in the correct place.

#### 3.3. Epicardial imaging

In order to complete this analysis and be able to make a reliable prognosis on the location of an ectopic focus we performed epicardial reconstructions, determined an optimal solution as before based on the L-curve criterium and visualized the results (see Fig. 1, third line). As expected, we obtained a good reconstruction of the epicardial extrasystole. For its endocardial counterpart we received a solution that is projected to the epicardium and has an inverted sign of the reconstructed signal. For the septal ectopic foci we did not get smooth reconstructions as in the case of Tikhonov regularization: a distinct dipolar structure of the TMV was observed in this case.

#### 3.4. Endo- and epicardial imaging with local weights

In case we know the amount of noise  $\epsilon$  in the system, we can use a discrepancy principle for calculating the optimal

regularization parameters  $(\lambda_1, \lambda_2)$  that produce a solution satisfying

$$\|Ax - y\| \leq \epsilon \quad (3)$$

Based on this criterium we solved the problem (2) for the parameters  $(\lambda_1, \lambda_2)$  and determined their optimal combinations (the results are shown in Fig. 1 in the last line). For the epicardial extrasystole, both  $\lambda_1$  and  $\lambda_2$  were equal to the regularization parameter from the Tikhonov reconstruction. For the endocardial events  $\lambda$  regularizing the endocardial surface were 2 orders of magnitude (100 times) smaller than those for the epicardium, i.e. larger spatial gradients on the endocardial surface were allowed.

#### 4. Discussion and conclusions

In this work we examined reconstructions of ventricular ectopic foci with the ECG imaging technique. A big challenge in cardiology is to distinguish between endocardial and epicardial events and between the left and right septal parts of the endocardium. For the endocardial extrasystoles a standard Tikhonov approach tends to push the components with large amplitudes to the epicardium in such a way that the original TMV gradient strength is preserved. In order to overcome this problem and reliably estimate the origin of an extrasystole we proposed to perform two additional reconstructions for exclusively endocardial or epicardial nodes. When thinking in terms of simple dipoles, the constraints on a false surface, e.g. solving the problem for the endocardial nodes while in reality it was an epicardial focus, the regularization should invert the sign of the signal in order to achieve the same dipole direction and produce a BSPM 'close' to the measured one (this is valid for the cases when an extrasystole did not become transmural). This simple observation allows us to easily classify ventricular wall extrasystoles: if both Tikhonov and epicardial imaging produce positive TMV on the epicardium and endocardial imaging produces a mirror-symmetrical negative signal on the endocardium, then we clearly identified an epicardial focus. For the cases of an endocardial focus Tikhonov and epicardial imaging will produce a sign-inverted solution on the epicardium while the endocardial reconstruction delivers a reasonable solution.

In cases of a septal extrasystole the Tikhonov solution as well as epicardial imaging would lead to positive TMV values on the epicardium projected from the septum. The endocardial imaging would correctly identify the location of the extrasystole in the septum and not on the outer heart wall endocardium as in the previously considered cases.

Review of three regularized solutions (Tikhonov, endocardial and epicardial imaging) enables us to find the origin of an ectopic focus (endocardium/epicardium) in question. When the information about the data misfit is available the method with different regularization weights for endocar-

dial and epicardial surfaces should be used.

#### References

- [1] Schilling RJ, Peters NS, Davies DW. Simultaneous endocardial mapping in the human left ventricle using a noncontact catheter. *Circulation* 1998;98:887–898.
- [2] Wang Y, Cuculich PS, Zhang J, Desouza KA, Vijayakumar R, Chen J, Faddis MN, Lindsay BD, Smith TW, Rudy Y. Noninvasive electroanatomic mapping of human ventricular arrhythmias with electrocardiographic imaging. *Science Translational Medicine* 2011;3(98).
- [3] Sapp J, Dawoud F, Clements J, Horáček B. Inverse solution mapping of epicardial potentials quantitative comparison with epicardial contact mapping. *Circulation Arrhythmia and Electrophysiology* 2012;5(5):1001–1009.
- [4] Nielsen BF, Lysaker M, Grottum P. Computing ischemic regions in the heart with the bidomain model; first steps towards validation. *Medical Imaging IEEE Transactions on* 2013;32(6):1085–1096.
- [5] Erem B, Coll-Font J, Martinez Orellana R, Stovicek P, Brooks D. Using transmural regularization and dynamic modeling for non-invasive cardiac potential imaging of endocardial pacing with imprecise thoracic geometry. *Medical Imaging IEEE Transactions on* 2013;33(3):726–738.
- [6] Schulze WHW, Potyagaylo D, Dössel O. Predicting the accuracy of cardiac pace mapping from lead field matrix characteristics. In *Proc. 40th International Congress on Electrocardiology*. 2013; .
- [7] Kohler T, Wagner M, Fuchs M, Wischmann H, Drenckhahn R, Theissen A. Depth normalization in meg/eeg current density imaging. In *Engineering in Medicine and Biology Society, 1996. Bridging Disciplines for Biomedicine. Proceedings of the 18th Annual International Conference of the IEEE*, volume 2. 1996; 812–813.
- [8] Sekihara K, Sahani M, Nagarajan S. Localization bias and spatial resolution of adaptive and non-adaptive spatial filters for meg source reconstruction. *NeuroImage* 2005; 25(4):1056–1067.
- [9] Simms HDJ, Geselowitz DB. Computation of heart surface potentials using the surface source model. *J Cardiovasc Electrophysiol* 1995;6(7):522–531.
- [10] Tikhonov AN, Arsenin VY. *Solutions of ill-posed problem*. Winston&Sons, New York, 1977.
- [11] Brooks DH, Ahmad GF, MacLeod RS, Maratos GM. Inverse electrocardiography by simultaneous imposition of multiple constraints. *IEEE Trans Biom Eng* 1999;46:3–18.
- [12] Hansen PC. *Computational inverse problems in electrocardiography*. WIT Press, *Advances in Computational Bioengineering*, 2001; 119–142.

Address for correspondence:

Danila Potyagaylo  
Kaiserstr. 12, 76128 Karlsruhe, Germany

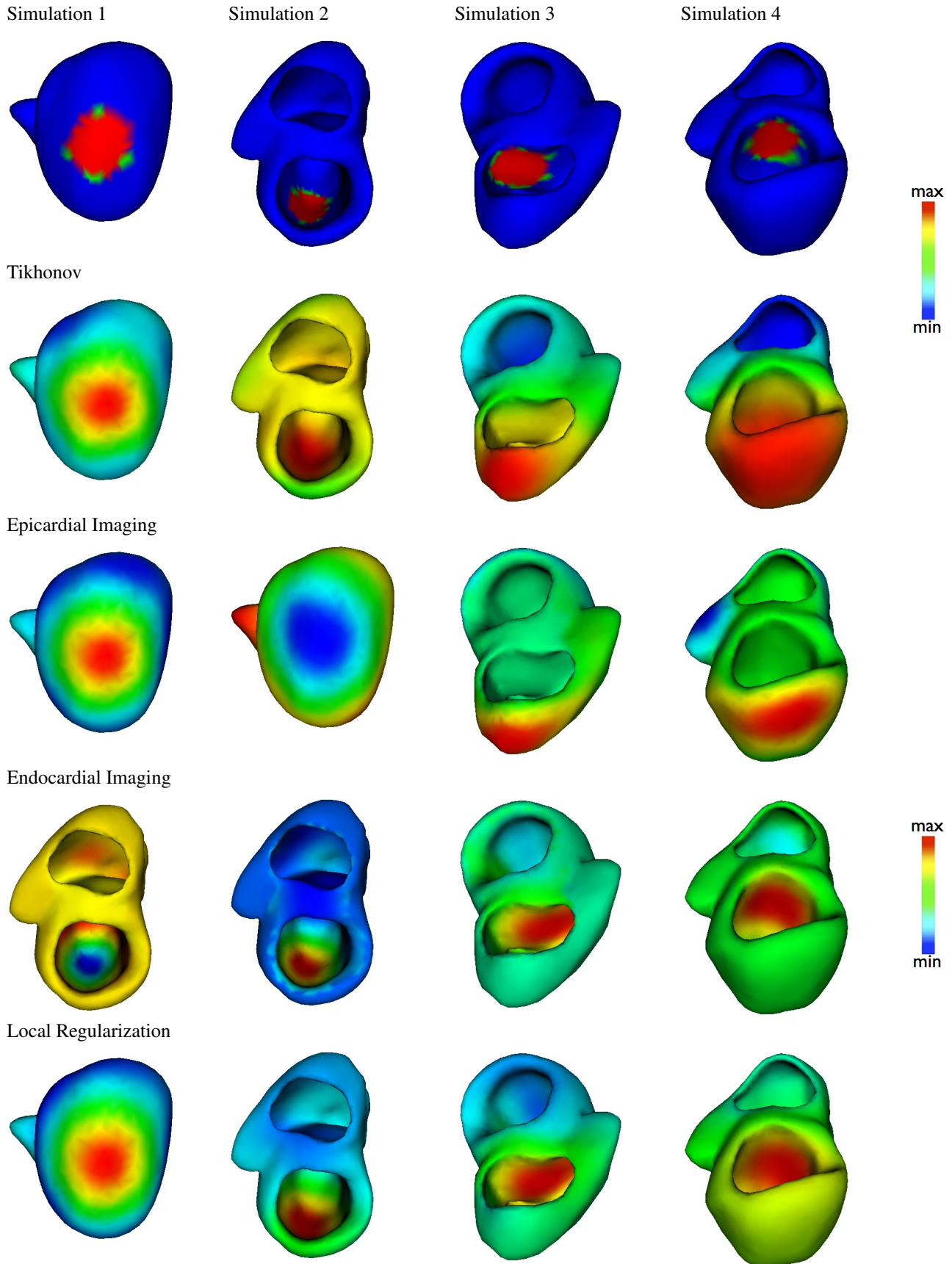


Figure 1: Simulations and reconstruction results for the TMV (mV) from different reconstruction methods.