Do we need to enforce the homogeneous Neuman condition on the Torso for solving the inverse electrocardiographic problem ?

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Abstract

Robust calculations of the inverse electrocardiographic problem may require accurate specification of boundary conditions at the torso and cardiac surfaces. In particular, the numerical specification of the no-flux condition on the torso is difficult because surface normals must be computed, and because the torso may alternatively be considered infinitely far away from the heart. Using the method of fundamental solutions (MFS) proposed in [1], this boundary conditions can be taken into account in different manners. Specifically, the no-flux condition on the torso can be ignored, or weighted with respect to the Dirichlet boundary condition associated to the torso data, or can be strongly enforced through a saddle-point problem. In this article, we compare these approaches.

In this work we provide a preliminary comparison of these different strategies.

1. Introduction

The epicardial heart potentials ϕ_E are classically related to the torso potentials ϕ_T in a quasi-static manner (see e.g. [2]) by solving the electrostatic problem in the volume conductor Ω enclosed by the heart epicardial surface Γ_E and the body surface Γ_T, with a no flux condition on the body surface and a given dataset ϕ_E on the epicardial surface,

$$-\operatorname{div}(\phi) = 0, \text{ in } \Omega, \tag{1}$$

$$\partial_n \phi = 0 \text{ on } \Gamma_T, \quad \phi = \phi_E \text{ on } \Gamma_E. \tag{2}$$

The body surface potential map $\phi_T = \phi$ on Γ_T can be finally retrieved from the unique solution to this problem.

The inverse electrocardiographic problem consists in finding a epicardial map ϕ_E from the knowledge of a body surface map ϕ_T. It can be formulated as the well-known ill-posed problem

$$-\operatorname{div}(\phi) = 0, \text{ in } \Omega, \tag{3}$$

$$\partial_n \phi = 0 \text{ on } \Gamma_T, \quad \phi = \phi_T \text{ on } \Gamma_T. \tag{4}$$

There are lots of possible approaches to find an interesting solution to this problem. Furthermore, the discretization of the problem is of great importance in finding a relevant solution. We will focus on the method of fundamental solution associated to a Tickonov regularization as proposed by Y. Rudy and collaborators in [1]. It consist representing a function harmonic in the domain Ω (i.e. that solves (3)) as the electrostatic field associated to a finite number of pointwise electrical sources (i.e. Dirac masses) located strictly outside of Ω. Specifically the potential ϕ for $x \in \Omega$ is search as $\phi(x) = a_0 + \sum_{i=1}^{N_S} f(x - y_i)a_i$, where the $(y_i)_{i=1\ldots N_S}$ are the N_S locations of the sources ($y_i \notin \Omega$), and the $(a_i)_{i=1\ldots N_S}$ are their amplitudes. The potential is defined up to a constant, given by the first source a_0 (see [1]). Here, f stands for the fundamental solution to the Laplace equation in $\mathbb{R}^3$, specifically $f(r) = \frac{1}{4\pi}\frac{1}{|r|}$ for all $r \in \mathbb{R}^3$. Hence, solving the complete inverse problem amounts to look for some locations (y_i) and sources (a_i) such that ϕ verifies as well as possible the two boundary conditions (4). In practise, the loactions are considered fixed, and only the sources are unknows, resulting in a quadratic minimizing problem: find the sources $a = (a_0, a_1 \ldots a_{N_S})^\top \in \mathbb{R}^{1+N_S}$ that minimize $J(a) = \|\phi - \phi_T\|^2 + \|\partial_n\phi\|^2 + \alpha|a|^2$.

After discretization on collocation points on the boundary Γ_T, this formulation yields a linear system of equations for the sources a. The linear system involves contributions

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of the Dirichlet conditions, and the Neuman condition, in an apparently equivalent manner. The coefficients of the numerical system are exactly the values taken by the fundamental solution and its normal derivatives at the colocation points. Numerically, the Neuman condition requires to build the normal to the body surface, which is a first difficulty. In addition, the body is cut at the top and bottom of the torso and the arms, where no-flux conditions are probably not relevant. Lastly, the coefficient related to the Neuman condition appear to be much smaller than the one from the Dirichlet condition, due to the distance between the torso and the actual electrical source (the heart).

These remarks suggest to consider again the role and discretization of the Neuman boundary condition when solving for the inverse electrocardiographic problem. We will consider the family of problems in which the norms $\|\phi - \phi_T\|^2$ and $\|\partial_n \phi\|^2$ are weighted in the cost function $J(a)$, and a problem in which the Neuman condition is strongly ensured by taking $\partial_n \phi = 0$ on Γ_T as a constraint in the minimization problem. These problems will be compared. The results will be evaluated by using datasets obtained with an anatomically and electrophysiologically accurate computer model [3] (based on clinical images).

2. Methods

2.1. General setup

The volume conductor Ω representing the body is supposed to have an interior closed boundary Γ_E (epicardium) on which N_E points $(x_i^E)_{i=12\dots N_E}$ are known, and an exterior closed boundary Γ_T (body surface) represented by N_T points $(x_i)_{i=1\dots N_T}$. We also assume that N_T vectors $(n_i)_{i=1\dots N_T}$ normal to the body surface at each x_i are available. Like in [1] we use $N_S = N_T + N_E$ sources located on both sides of the body volume, at points $(y_i)_{i=1\dots N_S}$, with N_S coefficients $(a_i)_{i=1\dots N_S}$. The locations are actually obtained by deflating Γ_E and inflating Γ_T. Given a potential function $\phi(x) = a_0 + \sum_{i=1}^{N_S} f(x - y_i)a_i$, we gather its boundary value at the N_T points x_i on the body surface in the vector $\phi_T = (\phi_1 \dots \phi_{N_T})^\top = B_0 a$, and the values of its normal derivatives in the vector $\partial_n \phi_T = (\partial_{n_1}\phi_1 \dots \partial_{n_{N_T}}\phi_{N_T})^\top = B_1 a$. Here B_0 and B_1 are $N_T \times (1 + N_S)$ rectangular matrices defined by

$$B_0 = \begin{pmatrix} 1 & f(r_{11}) & \dots & f(r_{1N_S}) \\ \vdots & \vdots & & \vdots \\ 1 & f(r_{N_T 1}) & \dots & f(r_{N_T N_S}) \end{pmatrix}, \quad (5)$$

$$B_1 = \begin{pmatrix} 0 & \partial_{n_1} f(r_{11}) & \dots & \partial_{n_1} f(r_{1N_S}) \\ \vdots & \vdots & & \vdots \\ 0 & \partial_{n_{N_T}} f(r_{N_T 1}) & \dots & \partial_{n_{N_T}} f(r_{N_T N_S}) \end{pmatrix}, \quad (6)$$

where $r_{ij} = x_i - y_j$ for $i = 1 \dots N_T$ and $j = 1 \dots N_S$.

2.2. The usual minimization problem

Given some torso data $\phi_T^* = (\phi_i^\star)_{i=1\dots N_T}$ (e.g. measured or built *in silico*), the usual problem as defined in [1] consists in choosing $a \in \mathbb{R}^{1+N_S}$ that minimizes

$$J(a) = \frac{1}{2}\left|B_0 a - \phi_T^\star\right|^2 + \frac{1}{2}\left|B_1 a\right|^2 + \frac{1}{2}\alpha|a|^2, \quad (7)$$

where $\alpha > 0$ is a Tikhonov regularization parameter. In practise, it is obtained by using the CRESO technique on a time sequence of data. Minimizing J amounts to approximately solve the block linear system

$$\begin{pmatrix} B_0 \\ B_1 \end{pmatrix} a = \begin{pmatrix} \phi_T^\star \\ 0 \end{pmatrix}. \quad (8)$$

Once a is determined, the potential on the epicardial surface $\phi_E = (\phi(x_i^E))_{i=1\dots N_E}$ are obtained by applying the definition $\phi(x_i^E) = a_0 + \sum_{j=1}^{N_S} f(x_i^E - y_j)a_j$.

2.3. Minimization with weighed boundary conditions

In practical cases, we observed that $\|B_1\| \ll \|B_0\|$ (using e.g. the spectral norm). It means that the Neuman boundary condition is under-considerated in the resolution.

Hence, we introduce the weighted cost function

$$J_\lambda(a) = \frac{(1-\lambda)^2}{2}\left|B_0 a - \phi_T^\star\right|^2 + \frac{\lambda}{2}\left|B_1 a\right|^2 + \frac{1}{2}\alpha|a|^2, \quad (9)$$

where λ is a weight. The corresponding linear system is

$$\begin{pmatrix} (1-\lambda)B_0 \\ \lambda B_1 \end{pmatrix} a = \begin{pmatrix} \phi_T^\star \\ 0 \end{pmatrix}. \quad (10)$$

Note that for $\lambda = 0$ the Neuman boundary conditions are completely neglected, while for $\lambda = 0.5$, the usual problem is recovered.

Finally, the boundary conditions will have the same importance in the numerical problem if we choose λ such that $\|(1-\lambda)B_0\| = \|\lambda B_1\|$. If we restrict to $\lambda < 1$ and define $\epsilon = \frac{\|B_1\|}{\|B_0\|} \ll 1$, then $\lambda = \frac{1}{1+\epsilon}$ provides that the boundary conditions are considered equivalently.

2.4. Minimization with the Neuman condition as a constraint

Finally, the Neuman condition can be introduced as a homogeneous equality constraint. In this case, we look for $a \in \mathbb{R}^{1+N_S}$ that minimizes

$$J_0(a) = \frac{1}{2}\left|B_0 a - \phi_T^\star\right|^2 + \frac{1}{2}\alpha|a|^2, \quad (11)$$

under the constraint that $B_1 a = 0$ in $\mathbb{R}^{N_T}$. This is a saddle-point problem. We can introduce its Lagrangian

$$L(a,\mu) = \frac{1}{2}\left|B_0 a - \phi_T^\star\right|^2 + \frac{1}{2}\alpha |a|^2 + \mu^\top B_1 a, \qquad (12)$$

where $\mu \in \mathbb{R}^{N_T}$ is a Lagrange multiplier. The solution is found by solving the block linear system

$$\begin{pmatrix} B_0^\top B_0 + \alpha\,\mathrm{Id} & B_1^\top \\ B_1 & 0 \end{pmatrix} \begin{pmatrix} a \\ \mu \end{pmatrix} = \begin{pmatrix} B_0^\top \phi_T^\star \\ 0 \end{pmatrix}. \qquad (13)$$

This system has a unique solution provided $\mathrm{span}(B_1) = \mathbb{R}^{N_T}$, and because $B_0^\top B_0 + \alpha\,\mathrm{Id}$ is positive-definite due to the Tikhonov regularization.

We actually solve this problem with SDPT3, a primal-dual path-following algorithm available on the CVX package for convex optimization [4].

2.5. Comparisons

We compared the matrices B_0 and B_1, and compared the signals reconstructed by the minimization technique without Neuman boundary condition ($\lambda = 0$ in (9)), the usual minimization technique ($\lambda = 0.5$), the minimization with balanced contributions of the boundary conditions ($\lambda = 0.9987$, see section 3.1), and the constrained minimization technique from section 2.4. Several datasets were simulated using a realistic anatomically shaped human model, one single site pacing (LV lateral endocardial – lasts 500 ms), and 4 single spiral waves with different conductivity coefficients (last 3000 ms). The simulations provide both the theoretical, *in-silico* epicardial potential map $\phi_E^\star$ and torso data $\phi_T^\star$ every 1 ms. The reconstructed ϕ_E were compared to simulated ones. Correlation coefficients (CC) and relative errors (RE) for both potential maps and electrograms were calculated exactly like in [1].

3. Results

3.1. Comparison between the Dirichlet and Neuman matrices

We calculated the ratio $\epsilon = \|B_1\|/\|B_0\|$ defined in section 2.3 on the geometry used for the simulations. This ratio depends only on the geometry and location of the sources (for which we use a fixed rule).

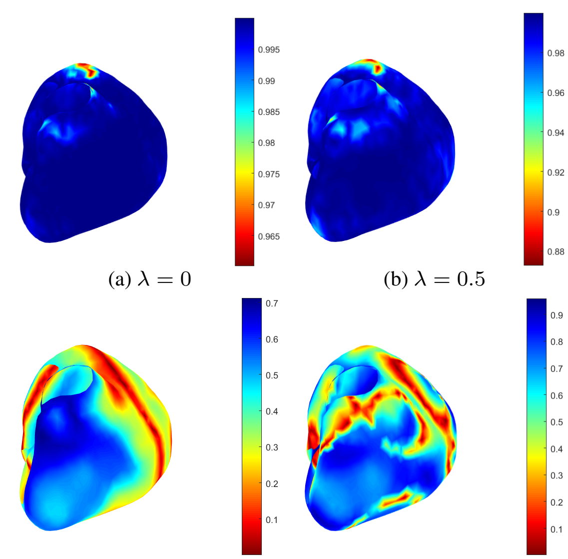
We find that $\epsilon = 0.0013$. The regularization coefficient computed following the CRESO rule was $1.e-3 \leq \alpha \leq 1.e-2$ for the datasets studied here. As a conclusion, the Neuman condition contributes to the classical ($\lambda = 0.5$) resolution of the inverse problem in a negligible manner. Instead, the choice $\lambda = 1/(1+\epsilon) = 0.9987$ restores the balance between the contributions of the Neuman and Dirichlet conditions in the cost function J_λ (eq. (9)).

3.2. Comparison between the reconstructions

We reconstructed the epicardial potentials for the five activation sequences mentionned above, using the minimization problem (9) for $\lambda = 0$ (no Neumann condition), $\lambda = 0.5$ (usual case), and $\lambda = 0.9987$ (the contributions of the boundary conditions balance each other). The Tikhonov regularization parameter was always chosen by the CRESO rule, and ranges between $1e-3$ and $1e-2$.

We also reconstruced the activation sequences using the constrained minimization problem (11). The Tikhonov regularization parameter was searched between $1.e-5$ and $1.e-1$, and the value obtained for the corresponding dataset with the unconstrained problem ($\lambda = 0.5$). In all cases, the Lagrange multipliers μ were significantly higher on the electrodes on the front of the torso than on the ones on the back. The multipliers for the electrodes on the back are about 100 times smaller the the largest mutliplier. Indeed the front electrodes are closer to the heart surface than the back ones.

Figure 1 and table 1 shows the CC and RE for the electrograms for each of these four reconstructions for the single pacing site simulation. In addition, a statistical analyis



(a) $\lambda = 0$ (b) $\lambda = 0.5$ (c) $\lambda = 0.9987$ (d) Constrained minimization

Figure 1: CC maps for electrograms, single site pacing.

was carried out on the 5 datasets available. The resolution with $\lambda = 0$ and $\lambda = 0.5$ were compared using paired t-test for normally distributed data and a Wilcoxon signed ranked test for non-normally distributed. Normality was determined with a Shapiro-Wilk test on the difference between paired data. We found that the CC and RE maps for electrograms were not normally distributed, and that the case $\lambda = 0$ has significantly higher CC ($p < 1.e-4$), and

	min	average	max
$\lambda = 0$	1.38e-5	0.0643	17.2
$\lambda = 0.5$	3.13e-5	0.171	19.4
$\lambda = 0.9987$	5.60e-2	1.01	7.03
constrained minimization	4.3e-3	3.56	347

Table 1: RE for electrograms, single site pacing.

lower RE ($p < 1.e-4$) than the case $\lambda = 0.5$.

4. Discussion and conclusions

The classical formulation of the ECGI inverse problem with the MFS method [1] involves a linear system. Its matrix can be split into two matrices B_0 and B_1, that correspond to the Dirichlet and Neumann boundary condition of the problem, respetively. We observed in section2.3 that the ratio between the norms of these matrices is $\epsilon = 0.0013$. Hence the Neumann boundary condition contributes weakly to the global resolution. Consequently, we wondered wether this condition should be considered more strongly in the problem, or avoided.

We introduced a weighting coefficient $0 \leq \lambda \leq 1$ in the cost function (section 2.3), so as to change the balance between the contributions of the Neumann and Dirichlet conditions. For $\lambda = 0$, the Neumann condition is ignored, while for $\lambda = 0.9987$ both conditions have the same importance. In addition, we propose to consider the Neumann condition as a constraint, so that it is resolved exactly.

Reconstructions of potentials are computed for datasets from a numerical model (*in-silico* data), and compared to the original modeling epicardial data. The results can be split in two parts, as expected: for $\lambda \in \{0, 0.5\}$, (weak or no Neumann condition at all) on one side, and for $\lambda = 0.9987$ or the constrained minimization (strong or exact Neumann condition) on the other side. The results for no Neumann condition are in better agreement with the original data (fig. 1, table 1, and the statistical data from section 3.2).

Hence the ECGI inverse problem may be solved with the MFS method by looking only for sources that match the Dirichlet boundary condition (miniizing $J_0(a) = \frac{1}{2}\left|B_0 a - \phi_T^\star\right|^2 + \frac{1}{2}\alpha|a|^2$), while the Neumann condition is assumed to be respected by default.

The MFS method without Neumann condition has a reduced computational cost, since the size of the linear system is divided by 2. Note also that its condition number is largely reduced, so that it is less sensible to the choice of the regularization parameter. In addition, the normal vectors to the torso boundary are not necessary anymore.

Naturally, these ideas regarding the Neuman condition can be explored further in order to improve the resolution of the ECGI problem. For instance, the artificial cuts at the endings of the torso may be better taken into account, by appropriate boundary conditions. Furthermore, the Lagrange multipliers μ from the constraint approach may provide additional information on the importance of the Neumann condition at individual electrodes. They could be useful to refine the inverse solution. Finally, we used only 5 ventricular datasets obtained by a numerical model. The case of complex atrial activations, and clinical data must be considered for a complete evaluation of the techniques proposed.

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