Description of the Volume-Clamp Method of Blood Pressure Measurements Using the Mathematical Model of the Lamé Problem

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Abstract

The volume-clamp method allows non-invasive continuous blood pressure monitoring from a finger, however this method lacks a solid model describing the physical phenomena during volume clamp. This paper proposes a simple model using the concept a thick-walled cylinder loaded with inner (blood pressure) and external (cuff pressure) pressures – the Lamé cylinder.

The simulation of method calibration i.e. finding the correct pressure value of operating point and blood pressure measurement was performed based on Lamé equation for a thick cylinder.

It was found that in this model, wall stress is not to be neglected, even when blood pressure equals cuff pressure. The elastic properties of the artery wall are crucial for finding the correct operating point. Maximum volume oscillation occurs when transmural pressure fits the minimum of Young's modulus. For a constant modulus, calibration of the method is impossible. The reverse situation occurs for blood pressure simulation: for constant modulus, a linear characteristic was found. This result suggests that finger physical properties, especially artery elasticity, may disturb measurement of blood pressure even when transmural pressure is 0.

1. Introduction

In 1967 Penaz patented the device for continuous non-invasive blood-pressure monitoring [1]. Four years later, a working device, using finger cuff, was shown at a conference in Dresden [2]. During measurement, the device maintained constant finger volume by changing cuff pressure (the volume-clamp method). In this method, the blood pressure and cuff pressure were assumed to be equal during the volume clamp. Mechanical properties, especially the elasticity of a finger, were neglected since the volume did not change.

To correctly measure the blood pressure, correct set-point of cuff pressure must be found. Different criteria for finding this point have been proposed [3]. Wesseling et al. [4] developed an algorithm for automatic determination of the operating point and patented the method for correcting its changes in time.

In their method, the volume of the finger is measured by photoplethysmography – as degree of light absorption. Changes in the light absorption are assumed to be proportional to changes in finger volume [5].

The volume-clamp method was validated with the gold standard techniques. Wesseling et al. [6] estimated the mean error of the measured blood pressure as 9-16% with respect to the Korotkoff method and 6-7% with respect to invasive measurement.

The volume-clamp method lacks a solid model that can describe the phenomena during continuous non-invasive blood pressure measurement using the finger cuff. As a model we employ is a thick-walled cylinder loaded with inner (blood pressure) and external (cuff pressure) pressures – the Lamé cylinder [7]. A simulation of a processes of finding the correct pressure value of operating point and blood pressure measurement was performed. The impact of wall properties on the accuracy of these processes will be examined.

2. Method

2.1.Thick-walled cylinder as the finger model

During measurement using the volume-clamp method, the artery wall is loaded with two pressures: inner blood pressure and external cuff pressure. The simplest description of finger geometry is a thick-walled cylinder – the inner radius represents the diameter of the artery, the external radius corresponds to finger diameter (Figure 1). To calculate stress and distortions, the Lamé equation for a thick cylinder was used [8].

In the model, the thick-walled cylinder (with the inner (r_i) and external (r_e) radius) is loaded with external (P_e) and internal (P_i) pressures that cause shear stress (σ_Θ) and radial stress (σ_r), Figure 1A. Axial symmetry of the finger is assumed. According to Hooke's law stress causes displacements (u) of the wall. Pressure load results in change of the external radius r_e by $u(r_e)$ and internal radius r_i by $u(r_i)$ (Figure 1B).

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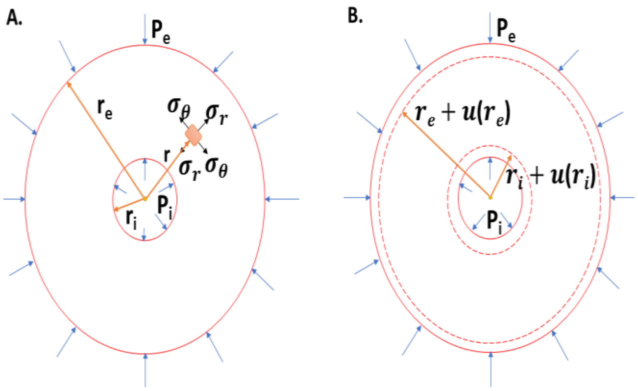

Figure 1: A. thick-walled cylinder (with the inner (r_i) and external (r_e) radius) is loaded with external (Pe) and internal (P_i) pressures that cause shear stress (σ_Θ) and radial stress (σ_r). Stresses cause deformation (u) that changes the geometry of the cylinder.

Wall radius stress (σ_r) and shear stress (σ_Θ) in the thick-walled cylinder are equal to [9]:

$$\sigma_r = \frac{P_i r_i^2 - P_e r_e^2}{r_e^2 - r_i^2} - \frac{r_i^2 r_e^2 (P_i - P_e)}{r_e^2 - r_i^2} * \frac{1}{r^2} \quad (1)$$

$$\sigma_\theta = \frac{P_i r_i^2 - P_e r_e^2}{r_e^2 - r_i^2} + \frac{r_i^2 r_e^2 (P_i - P_e)}{r_e^2 - r_i^2} * \frac{1}{r^2} \quad (2)$$

Where: P_i – the pressure inside the cylinder corresponding to blood pressure in the finger arteries; P_e – pressure outside the cylinder, equal the cuff pressure; r_i – cylinder inner radius – summarized radius of all finger's arteries; r_e – cylinder external radius – radius of the finger; r – given radius for stress calculation.

As an effect of the stresses, the deformation (u) – changes the wall radius. The deformation is described by formula below:

$$u(r) = \frac{1}{E({r_e}^2 - {r_i}^2)}[(1-\nu)(P_i {r_i}^2 - P_e {r_e}^2)r +$$
$$+(1+\nu){r_i}^2{r_e}^2(P_i - P_e)\frac{1}{r}] \quad (3)$$

Where: E – Young's modulus of cylinder wall; ν – Poisson's ratio of cylinder wall.

2.2. Young's modulus and Poisson's ratio of the artery wall

Young's modulus for artery walls can be calculated from the pressure-volume curve measured in vitro. Varies values can be found in the literature. Young's modulus can be assumed to depend on pressure in the artery or to have a constant value, not dependent on intra-arterial pressure.

Zhou and Fung [10] found that Young's modulus of the intima-media layer of the pig thoracic aorta is 43.2 +/- 15.8 kPa. Hughes et al. [11] use the exponential function:

$$E = E_0 e^{(a*P)} \quad (5)$$

Where: E_0 = 667 mmHg and a = 0.017 [1/mmHg], this function being the best fit of the result obtained for the abdominal aorta of 12 dogs over a range of mean blood pressures from 40 – 200 mm Hg.

Assuming Young modulus being the function of transmural pressure, the difference between internal and external pressure is to be used [11,12]. In this simulation, we use the absolute value of transmural pressure:

$$E = E_0 e^{(a*abs\,(P_i - P_e))} \quad (6)$$

Poisson's ratio in the model measures the deformation of an artery in the shear direction to the radial direction of loading. Poisson's ratio, for the artery wall, is in the range of ν = 0,1-0,5 [13].

2.3. Light absorption in the cylinder model

In the volume clamp method, the volume of a finger is determined indirectly with the measure of light absorption. Change in the level of transmitted light is measured. Absorption of a medium (ABS) can be described with Lambert's law as:

$$ABS = log_{10}\frac{I_0}{I} \quad (7)$$

Where: I_0 – luminous flux incident on finger skin; I – the amount of light after passing the cylinder

Attenuation of light be a medium is a sum of the attenuation of its components:

$$ABS = \sum_i^n d_i * k_i \quad (8)$$

Where: d_i – thickness of i-th component; k_i – absorption coefficient for i-th component.

In the model proposed by us there are two components: a wall (with a diameter equal to 2*(R_e-R_i)) and blood in the artery (inner cylinder). The absorption coefficient k for artery wall for light (800 µm) is 0.04 [1/cm][14]. For the artery in this simulation, we assume the wall diameter as 10cm so absorption coefficient in the model is 0.4 [1/cm].

2.3. Analysis of the volume-clamp method with thick-walled cylinder

The model of the thick-walled cylinder is used to analyze the deformation of a finger by pressures of blood and cuff.. Changes in light absorption of a finger are analyzed for different cuff pressures within the assumed range of blood pressures during calibration process and during blood pressure measurement. We estimate cuff pressure necessary to compensate changes in light absorption caused by the change in blood pressure. Analysis was performed for constant Young's modulus (43.2 kPa), and Young's modulus as the exponential function of blood pressure (equation 6).

It was assumed that a finger is a homogeneous tissue with Poisson's ratio equal to 0.1. In every simulation, it was assumed that the inner diameter (diameter of arteries in finger) is equal to 0.1 cm, and the external diameter is equal to 1.4 cm (the diameter of an author finger).

3. Simulation results

3.1. Calibration of measurement – finding the

cuff pressure operating point

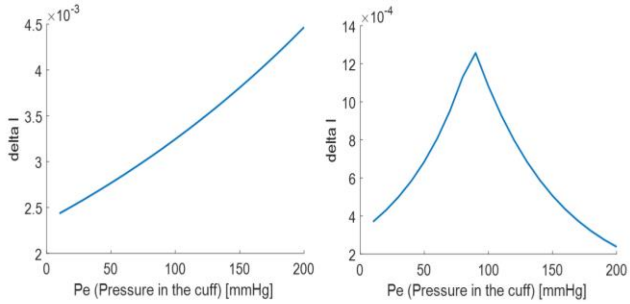

Figure 2: Plots present the impact of cuff pressure (from 0 to 200 mmHg) on the amplitude of changes in the amount of light after passing the cylinder. Left: for a constant wall Young's modulus. Right: for Young’s modulus as pressure function

Calibration of the volume-clamp method was simulated as finding the correct operating point, assumed to be a cuff pressure that allows the biggest finger volume oscillation, associated with a maximum difference in light transmission. In our simulation, blood pressure value was within the range of 80-120 mmHg. For a given cuff pressure (0-200mmHg), light transmission for a given condition was estimated. Light transmission amplitude as the difference between the maximum and minimum values (delta I) for given pressure was calculated. The results for fixed and pressure dependent Young's modulus are plotted in Figure 2. For constant Young’s modulus (left), there is no maximum in the delta I. An increase in cuff pressure causes an increase in delta I. For pressure-dependent Young's modulus, the maximum in delta I exists, and calibration can be performed.

3.2. Simulation of continuous blood pressure measurements

During measurement of blood pressure in the volume clamp method, the device tries to maintain fixed light transmission, thus finger volume by changing finger cuff pressure. The method assumes equality of blood pressure and cuff pressure at operating point found during calibration.

The measurement condition within a thick-walled cylinder model was simulated. It was assumed that operating point was P_e=P_i=100mmHg. Base light transmission for that condition (I_0) was calculated. Next, for different Pi, P_e that allows maintenance I_0 was sought. It was assumed that Pe must be in the range 0-200mmHg. The measurement characteristic for constant Young’s modulus is a linear function P_e (P_i).

For Young's modulus assumed to be pressure dependent, there is no valid measurement characteristic (Figure 3). For blood pressure lower than 100 mmHg, there is no P_e that can compensate for the changes in light transmission (Figure 4, B). For P_i greater than 100 mmHg, two P_e can provide maintained light transmission (Figure 4, A).

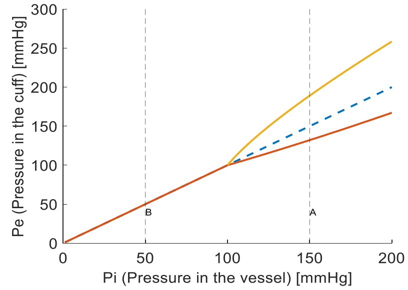

Figure 3: Measurement characteristic for pressure-dependent Young's modulus. P_e is the closest pressure necessary to maintain constant cylinder volume. The characteristic is divided into two areas.

Two valid states are possible for blood pressure greater than the given operating point (P_e=P_i=100 mmHg) (Figure 4, left). For P_i<100 mmHg, maintaining constant volume is impossible. (Figure 4, right).

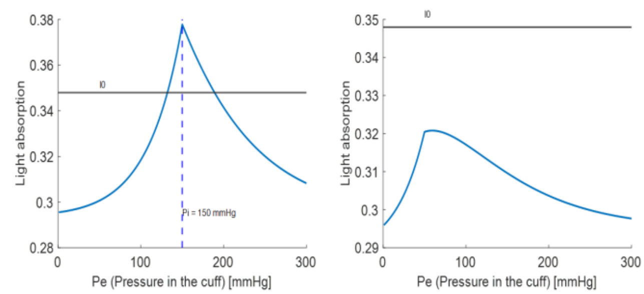

Figure 4: Left: For given P_i greater than operating point pressure here are two P_e values that allow the maintaining of a given light absorption level. Right: For Pi lower than operating point no P_e can compensate for the drop in light absorption.

4. Discussion

Imholz et al. [15] reviewed literature about NIBP. The authors summarize the results from 43 studies where Finapres was used. They determined method error as: for systolic pressure: -0,8 ± 11,7 mmHg, for diastolic pressure: -1,6 ± 7,7 mmHg, and mean pressure: -1,6 ± 8,5 mmHg. Wesseling et al. [6] state that the volume clamp method overestimates systolic pressure compared to Korotkoff, on average by six mmHg with a standard deviation of 20 mmHg. The method also underestimates diastolic pressure by three mmHg, with a standard deviation of 11 mmHg. Wesseling et al. [6] estimate that the mean measurement error with the volume clamp method is 6-16% with respect to non-invasive measurements and 6-7% compared to invasive measurements.

A significant standard deviation of measurement error was reported. Also, the temperature of the finger can affect measurements with the volume-clamp method. Tanaka and Thulesius [16] found that systolic blood pressure and finger temperature are inversely correlated.

The calibration algorithm of the volume clamp method is close to the oscillometric cuff blood pressure measurements. Chandrasekhar et al. [17] state that the search pressure associated with maximal cuff oscillation is the measurement of artery properties as a function of pressure. The maximum of that oscillation occurs at cuff pressure close to mean blood pressure, but it may differ from it, depending on artery elastic properties. Our simulations indicate that arteries' elastic properties, expressed as Young's modulus, are crucial to blood pressure measurement.

The limitations of the thick-walled cylinder model should be emphasized. The geometry used to describe the finger is a huge simplification. Also a finger is not homogeneous tissue. There are a bone, muscles, skin, etc., and each element has a different Young's modulus. The used value of artery wall Young's modulus as is a simplification. The model is only an illustration. The results obtained cannot be directly transferred to actual conditions

It is reasonable to look for better measurement algorithms for non-invasive cuff blood pressure measurements.

Acknowledgments

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