An Extension of Quadratic Variation Regularization for Simultaneous Baseline Wander and Power Line Interference Removal from ECG

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Abstract

Quadratic variation (QV) regularization is a recent method for BW removal but cannot be used for PLI cancellation. To overcome this limitation, an extension of QV regularization which can simultaneously deal with BW and PLI tracking and removal is presented. In the proposed method, PLI and BW are respectively modeled by a sinusoidal and a polynomial function. The difference equation of the sinusoidal function and the p-th order derivative of the polynomial function are then used as constrains in the optimization problem. The proposed approach is also implemented using Kalman filter and smoother which is an optimal estimator in mean square error (MSE) sense. We tested the method over data from the PhysioNet PTB database. Simulation results confirm the effectiveness of the approach and highlight its ability to simultaneously track and remove the PLI and BW.

1. Introduction

Recently, Fasano and Villani have proposed an approach to baseline wander (BW) estimation and removal for bioelectrical signals [1,2] which is based on the notation of quadratic variation (QV) reduction. In this approach, the BW is estimated using a constrained convex optimization problem where the first order derivative of the signal enters as a constraint. They showed the effectiveness of the QV regularization in BW removal from electrocardiogram (ECG) and electroencephalogram (EEG) using numerical examples. The smoothing approach defined by QV regularization [1, 2], is to recover an unknown signal $x(t)$ from its observation $y(t)$ in

$$y(t) = x(t) + v(t), \tag{1}$$

where $v(t)$ is an additive noise assumed to be uncorrelated with $x(t)$. An estimate of the signal of interest is obtained by solving the following least-square estimation (LSE) problem:

$$\hat{x}(t) = \underset{x(t)}{\operatorname{argmin}} \int [y(\tau) - x(\tau)]^2 \, d\tau + \lambda \int [D^p x(\tau)]^2 \, d\tau \tag{2}$$

where $D^p x = \frac{d^p}{dt^p} x$ denotes the p-th order derivative of the signal and λ denotes the regularization factor which is used to balance the the fidelity term (minimum mean square error) and signal smoothness. It is notable that in [1, 2], the first order derivative of the signal is considered (*i.e.*, $p = 1$) as constraint while Eq. (2) defines a general smoothing approach which penalizes the p-order derivative of the signal. In discrete-time domain, (2) can be written as follows [3]:

$$\hat{x}_k = \underset{x_k}{\operatorname{argmin}} \sum_{j=1}^{L} [y_j - x_j]^2 + \lambda \sum_{j=1}^{L} [\nabla^p x_j]^2 , \tag{3}$$

where $\nabla x_k = x_k - x_{k-1}$ is the first order difference and $\nabla^p x_k = \nabla(\nabla^{p-1} x_k)$ is the p-th order difference. In [3,4], we have shown that the value of λ is related to the cutoff frequency. The optimal solution for (3) is

$$\hat{x}_k = (\delta_k + \lambda \boldsymbol{b}_{p,-k} * \boldsymbol{b}_{p,k})^{\otimes} * y_k. \tag{4}$$

where $*$ and $\otimes$ are the convolution and deconvolution operator, respectively and $\boldsymbol{b}_p$ is defined by the following recursion:

$$\begin{cases} \boldsymbol{b}_1 \triangleq (+1 \ -1) & p = 1 \\ \boldsymbol{b}_p = \boldsymbol{b}_{p-1} * \boldsymbol{b}_1 & p > 1 \end{cases} \tag{5}$$

The frequency response of the p-th order QV regularization is

$$G_p(z) = \frac{1}{1 + \lambda(1 - z^{-1})^p (1 - z)^p}. \tag{6}$$

which becomes in the Fourier domain

$$G_p(e^{j\omega}) = \frac{1}{1 + \lambda[2 - 2\cos\omega]^p} = \frac{1}{1 + \lambda(2\sin\frac{\omega}{2})^{2p}}. \tag{7}$$

Therefore the value of λ is related to cutoff frequency as

$$\lambda = \frac{1}{(2\sin\frac{\omega_c}{2})^{2p}}, \tag{8}$$

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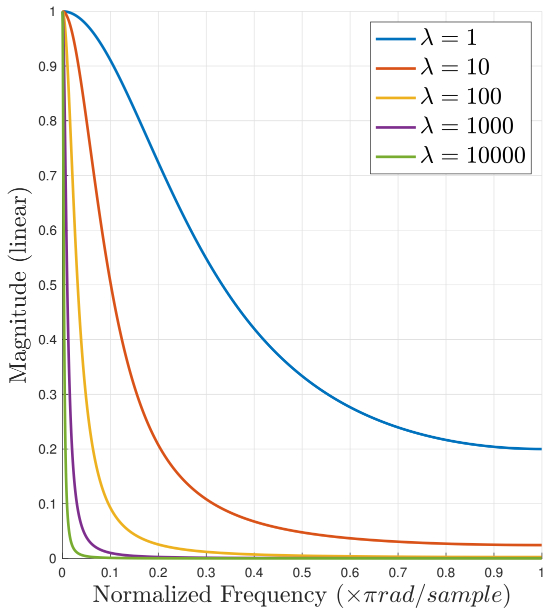


Figure 1. The frequency response of QV regularization for different values of λ.

where $\omega_c = 2\pi f_c$ and f_c is the cutoff frequency. The frequency response of the first order QV regularization for some low cutoff frequencies is plotted in Fig. 1. The QV regularization acts as a low-pass smoothing filter. Therefore, it can be used for BW tracking and removal from bioelectrical signals. However, the BW is not the only noise in measurement systems. For instance, power-line interference (PLI) is an unavoidable interference in biosignal measurement systems. As a limitation of QV regularization, it cannot be used for the estimation and removal of the PLI from bioelectrical signals. In this paper, we propose a modification to QV regularization which allows us to simultaneously track and remove the BW and PLI from bioelectrical signals.

2. Method

PLI and BW are respectively modeled by a sinusoidal and a polynomial function:

$$\begin{cases} x_p(t) = \alpha\cos(\omega_0 t + \phi) \\ x_b(t) = \sum_{i=0}^{p-1} \beta_i t^i \end{cases} \tag{9}$$

where $x_p(t)$ and $x_b(t)$ are PLI and BW, respectively, α, ω_0 and ϕ are the amplitude, frequency and phase of the PLI and $p-1$ is the order of polynomial used for BW modeling. The second order derivative of x_p is a function of itself and the p-th order derivative of x_b is zero:

$$\begin{cases} \frac{d^2}{dt^2} x_p(t) = -\omega_0^2 x_p(t) \\ \frac{d^p}{dt^p} x_b(t) = 0 \end{cases} \tag{10}$$

Since the above model is an approximate of the PLI and BW, the following differential equation model can be considered for PLI and BW tracking:

$$\begin{cases} \frac{d^2}{dt^2} x_p(t) + \omega_0^2 x_p(t) = w_p(t) \\ \frac{d^i}{dt^i} x_b(t) = w_b(t) \\ y(t) = x_p(t) + x_b(t) + v(t) \end{cases} \tag{11}$$

where $w_p(t)$, $w_b(t)$ and $v(t)$ are process and observation noises, respectively. The discrete-time version of (11) is:

$$\begin{cases} x_{p,k} = \gamma x_{p,k-1} - x_{p,k-2} + w_{p,k} \\ x_{b,k} = -\sum_{i=1}^{p} \alpha_i x_{b,k-i} + w_{b,k} \\ y_k = x_{p,k} + x_{b,k} + v_k \end{cases} \tag{12}$$

where $\gamma = 2\cos(\omega_0)$ and $\alpha_i = (-1)^i \binom{p}{i}$. (12) can be expressed in the following form:

$$\begin{cases} d_1(z) x_{p,k} = w_{p,k} \\ d_2(z) x_{b,k} = w_{b,k} \\ y_k = x_{p,k} + x_{b,k} + v_k \end{cases}, \tag{13}$$

where $z^{-i}x_k = x_{k-i}$, $d_1(z) = 1 - \gamma z^{-1} + z^{-2}$ and $d_2(z) = (1 - z^{-1})^p$. The difference equation of the sinusoidal function and the p-th order deference of the polynomial function are then used as constraints in the optimization problem:

$$\begin{aligned} \hat{x}_{p,k}, \hat{x}_{b,k} = \underset{x_{p,k}, x_{b,k}}{\operatorname{argmin}} & \sum_{j=1}^{L} \left(y_j - x_{p,j} - x_{b,j} \right)^2 \\ & + \sum_{j=1}^{L} \lambda_1 \left[d_1(z) x_{p,k} \right]^2 + \lambda_2 \left[d_2(z) x_{b,k} \right]^2 \\ & + 2\rho \left[d_1(z) x_{p,k} \right]^{\mathsf{T}} d_2(z) x_{b,k} \end{aligned} \tag{14}$$

where λ_1 and λ_2 are the regularization factors and ρ is the auto-correlation coefficient between w_p and w_b. Taking the derivative of (15) with respect to x_p and x_b and setting the results to zero, we find

$$\begin{cases} \hat{\boldsymbol{x}}_p = [\boldsymbol{M}_1\boldsymbol{M}_2 - \boldsymbol{N}_1\boldsymbol{N}_2]^{-1} \left[\lambda_1 \boldsymbol{D}^T\boldsymbol{D} - \rho\boldsymbol{D}^T\boldsymbol{B}_n\right] \boldsymbol{y} \\ \hat{\boldsymbol{x}}_b = [\boldsymbol{M}_1\boldsymbol{M}_2 - \boldsymbol{N}_1\boldsymbol{N}_2]^{-1} \left[\lambda_2 \boldsymbol{B}_n^T\boldsymbol{B}_n - \rho\boldsymbol{B}_n^T\boldsymbol{D}\right] \boldsymbol{y} \end{cases},$$

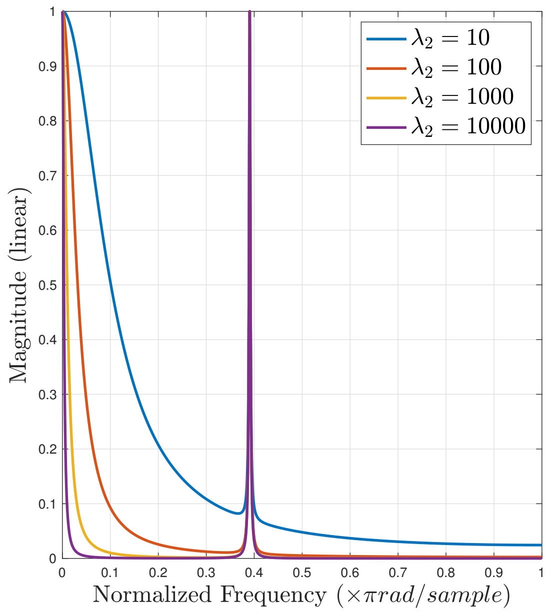


Figure 2. The frequency response of the proposed smoothing filter.

where

$$\begin{aligned}
\boldsymbol{M}_1 &= \boldsymbol{I} + \lambda_1 \boldsymbol{D}^T \boldsymbol{D}, \\
\boldsymbol{M}_2 &= \boldsymbol{I} + \lambda_2 \boldsymbol{B}_n^T \boldsymbol{B}_n, \\
\boldsymbol{N}_1 &= \boldsymbol{I} + \rho \boldsymbol{B}_n^T \boldsymbol{D}, \\
\boldsymbol{N}_2 &= \boldsymbol{I} + \rho \boldsymbol{D}^T \boldsymbol{B}_n,
\end{aligned}$$

$$\boldsymbol{D} = \begin{pmatrix} 1 & -\gamma & 1 & 0 & \dots & 0 \\ 0 & 1 & -\gamma & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -\gamma & 1 \end{pmatrix}$$

and

$$\boldsymbol{B}_n = \begin{pmatrix} 1 & \alpha_1 & \dots & \alpha_n & 0 & \dots & 0 \\ 0 & 1 & \alpha_1 & \dots & \alpha_n & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & \alpha_1 & \dots & \alpha_n \end{pmatrix}$$

The frequency response of the proposed smoothing filter is

$$\begin{aligned}
\psi(z) &= \frac{\hat{X}_p(z) + \hat{X}_b(z)}{Y(z)} = \frac{M_1(z) + M_2(z) - N_1(z) - N_2(z)}{M_1(z)M_2(z) - N_1(z)N_2(z)} \\
M_1(z) &= 1 + \lambda_1 d_1(z) d_1(z^{-1}), \\
M_2(z) &= 1 + \lambda_2 d_2(z) d_2(z^{-1}), \\
N_1(z) &= 1 + \rho d_2(\frac{1}{z}) d_1(z), \\
N_2(z) &= 1 + \rho d_1(\frac{1}{z}) d_2(z)
\end{aligned} \tag{15}$$

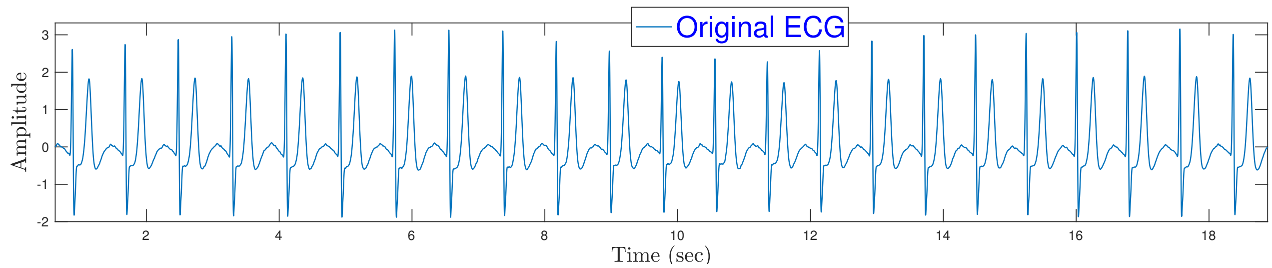
The frequency response of the proposed smoothing filter for $f_c = 0.5$ Hz and $f_p = 50$ Hz is plotted in Fig. 2. It allows the low frequencies and a narrow band-pass frequency band to pass. When the outputs are subtracted from the observation ($\hat{x}_k = y_k - \hat{x}_{p,k} - \hat{x}_{b,k}$), it acts as a simultaneous high-pass and a narrow-band notch smoothing filter. Finally, the proposed smoothing filter can also be implemented using a Kalman filter and a Kalman smoother. The idea has been recently presented in [5] and then extended in [6] for simultaneous linear time invariant (LTI) filtering and total variation (TV) denoising. In order to implement it, Eq. (12) can be used in the framework of Kalman filter and smoother and the states x_p and x_b are estimated through Kalman filter and smoother equations.

3. Results

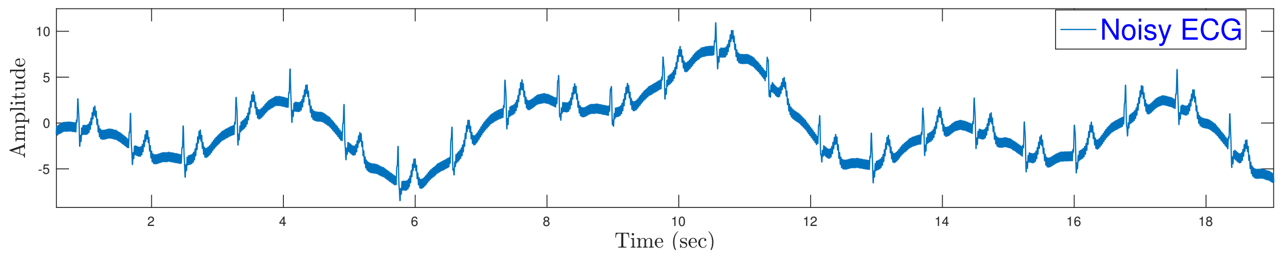
We applied the proposed smoothing filter to simultaneously estimate and remove the PLI and BW from ECG signals from PhysioNet PTB Diagnostic ECG Database [7], which contains 549 records from 290 subjects. Each record consists of twelve conventional ECG leads plus the three Frank's ones, sampled at 1kHz with 16-bit resolution. Figure 3 shows an example of a real ECG contaminated with synthetic PLI and BW. The original ECG and its noisy signal are respectively shown in Figure 3(a) and 3(b). The result of the proposed method is shown in Figure 3(c). It is seen that the PLI and BW are effectively removed using the proposed method.

4. Conclusion

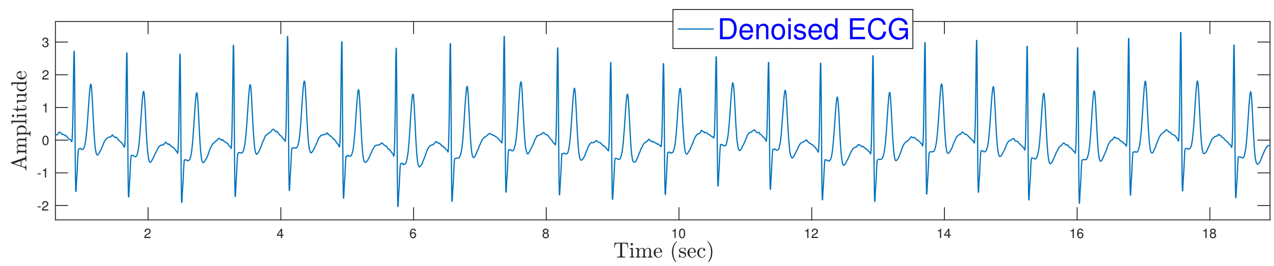
QV regularization is a low-pass smoothing filter which is suited for removing baseline wander but it cannot be used for powerline cancellation. This paper proposed an extension of QV regularization that makes it suited for simultaneous tracking and removal of BW and PLI. In the proposed approach, the PLI and BW are respectively modeled by a sinusoidal and a polynomial function. The difference equation of the sinusoidal function and the $p-1$-th order derivative of the polynomial function are then used as constrains in the optimization problem.



(a)Original ECG



(b)Deluded ECG by PLI and BW



(c)Denoised ECG using the proposed approach

Figure 3. Simultaneous track and removal of PLI and BW in ECG.

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