

Whole Heart Simulation of Severe Aortic Stenosis Using a Lumped Parameter Model of Heart Valve Dynamics

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Abstract

Lumped parameter models of the human circulatory system are able to reproduce major features and phases of human circulation. However, they often lack physiological detail regarding pressure and flow across the valves. To alleviate these shortcomings, we implement a model of heart valve dynamics based on Bernoulli's principle to account for the transvalvular pressure drop and extend it by smooth opening and closing of the valves. We evaluate the new model based on a simulation with healthy valves and explore the possibility of simulating heart valve diseases by considering a case of severe aortic stenosis. The model more faithfully reproduces pressure, volume, and flow in all four chambers and in particular across the valves. Most of the changes are related to the consideration of blood inertia. However, only by opening and closing the valves more slowly, it is possible to reproduce features connected to backflow. When reducing the maximum area ratio of the aortic valve to 10%, a pressure gradient of 77.2 mmHg during systole and a 20% reduction in stroke volume was observed in accordance with the AHA guidelines of severe aortic stenosis. To conclude, we were able to improve our existing 0D circulation model in terms of physiological accuracy by replacing the diode-like valves with an easy to implement model of heart valve dynamics that is capable of simulating both healthy and pathological scenarios.

1. Introduction

Certain cardiovascular diseases such as defective valves, stiffened blood vessels or increased vascular resistances influence the pumping efficiency and behavior of the heart. These influences can be studied in mathematical models of the heart if the underlying mechanisms between cardiac mechanics and circulation are considered sufficiently. Therefore, it is important to implement physiologically valid hemodynamic boundary conditions.

Previous publications that coupled 3D whole heart electromechanics to a 0D lumped parameter model of the circulatory system were able to achieve results in the phys-

iological range [1, 2]. However, both modeled the valves as diodes with an associated resistance. This makes it impossible to observe dynamics in flow and to include pathological conditions. A more complex valve model based on the effects of inertia due to acceleration in time and the Bernoulli effect was used by Augustin et al. [3]. Their valve opens instantly with a forward pressure drop and closes smoothly through a combination of backward pressure and forward flow. In a distributed modeling approach (1D), a model with a smooth opening and closing of the valves was used by Caforio et al. [4].

In this study, we extend our previously published circulatory system model [1] by a valve model derived from Bernoulli's equation [5, 6] and show that it is capable of reproducing physiological behavior in the case of healthy valves and a case of severe aortic stenosis.

2. Material and Methods

To study the effects of heart valve dynamics in whole heart simulations, we utilize the previously developed mesh shown in Figure 1. Since we are mainly interested in

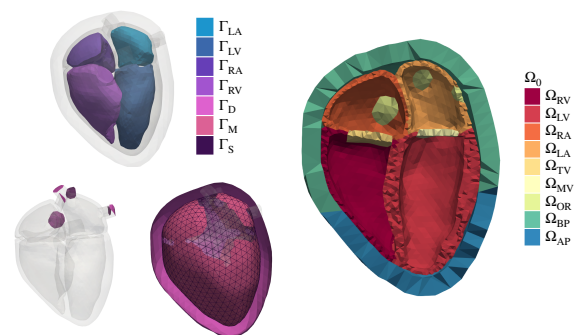


Figure 1. The computational domain of the diastolic configuration Ω with the boundaries Γ .

the effects of heart valve dynamics, we neglect electromechanical feedback mechanisms, i.e. our previous model

used in [1] reduces to

$$\begin{cases} \rho_0 \frac{\partial^2 \mathbf{d}}{\partial t^2} - \nabla \cdot \mathbf{FS} = \mathbf{0} & \text{in } \Omega_0, \\ \mathbf{FSN} = \mathbf{f}^{\text{contact}} & \text{on } \Gamma_0^{\text{Cl}}, \\ \mathbf{FSN} = -p_{\text{CH}} \mathbf{JF}^{-\text{T}} \mathbf{N} & \text{on } \Gamma_0^{\text{CH}}, \\ \mathbf{d} = \mathbf{0} & \text{on } \Gamma_0^{\text{D}}, \end{cases} \quad (1)$$

with the density ρ_0 , the displacement \mathbf{d} , the deformation gradient \mathbf{F} , its determinant $J = \det(\mathbf{F})$, the surface normal direction \mathbf{N} , and the 2nd Piola Kirchhoff stress $\mathbf{S} = \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}} + T_a \mathbf{f}_0 \otimes \mathbf{f}_0 (\mathbf{F} \mathbf{f}_0 \cdot \mathbf{F} \mathbf{f}_0)^{-0.5}$.

Since we only consider cardiac mechanics in this study, it is assumed that the development of active tension T_a is equal to the time course of chamber elastance $E(t)$. Stergiopoulos et al. [7] suggested that the elastance of the left ventricle can be approximated by two Hill functions:

$$e(t) = \frac{E(t) - E_{\min}}{E_{\max} - E_{\min}} = \frac{1}{k} \left(\frac{g_c}{1 + g_c} \right) \left(\frac{1}{1 + g_r} \right), \quad (2)$$

with $g_c = \left(\frac{t'}{\tau_c} \right)^{m_c}$, $g_r = \left(\frac{t'}{\tau_r} \right)^{m_r}$,

$$t' = \text{mod}(t - t_0, T), \quad k = \max(e(t)).$$

Equation (2) is used as an active stress driver function $T_a(t) = T_{\max} e(t)$ with the parameters in Table 1.

Table 1. Parameters for the active stress driver function.

Description	Ventricles	Atria
Contraction rate const. m_c	1.32	1.99
Relaxation rate const. m_r	14.5	11.2
Contraction time offset τ_c	0.215 s	0.042 s
Relaxation time offset τ_r	0.362 s	0.138 s
Onset time t_0	0.15 s	0.0 s
Period T	0.8 s	0.8 s
Peak tension T_{\max}	80 kPa	25 kPa

Myocardial tissue ($\Omega_{\text{RA}} \cup \Omega_{\text{LA}} \cup \Omega_{\text{RV}} \cup \Omega_{\text{LV}}$) is assumed to be hyperelastic, transverse isotropic, and nearly incompressible as described by [8] ($b_{\text{ff}} = 32.8$, $b_t = 13.1$, $b_{\text{ft}} = 22.9$, $C = 34.64 \text{ Pa}$, $\kappa = 650 \text{ kPa}$). The remaining tissue is modeled as a Neo-Hookean material.

To model the effect of the pericardium, we solve a frictionless contact handling problem [9] at the contact interface $\Gamma_{\text{Cl}} = \Gamma_{\text{M}} \cup \Gamma_{\text{S}}$. Additionally, omnidirectional spring boundaries were added at the distal end of the truncated pulmonary artery and the aorta with a spring stiffness of 1 MPa m^{-1} .

We calculate the pressure p_{CH} in the chambers $\text{CH} = \{\text{LA}, \text{RA}, \text{LV}, \text{RV}\}$ by coupling a closed-loop lumped parameter model of the circulatory system introduced in [1]

$$\frac{d\mathbf{c}(t)}{dt} = \mathbf{G}_c(t, \mathbf{c}(t), \mathbf{p}(t)) \quad \text{for } t \in (0, T] \quad (3)$$

to the heart via the volume-consistency constraint

$$V_{\text{CH}}^{\text{3D}}(\mathbf{d}(t)) = V_{\text{CH}}^{\text{0D}}(\mathbf{c}(t)). \quad (4)$$

All simulations are performed using CardioMechanics (<https://github.com/KIT-IBT/CardioMechanics>).

2.1. Model of Heart Valve Dynamics

We extend our previously published model of the human circulatory system [1] by a valve model proposed in [5]. It is based on a description of the instantaneous transvalvular pressure gradient Δp derived from the Bernoulli equation to express blood flow Q through the valves:

$$\frac{dQ}{dt} = \frac{\Delta p - B |Q| Q}{L} \quad \text{with} \quad (5)$$

$$B = \frac{\rho}{2 A_{\text{Eff}}^2} \quad \text{and} \quad L = \frac{6.28 \rho}{\sqrt{A_{\text{Eff}}}},$$

where the blood density $\rho = 1060 \text{ kg/m}^3$ and the effective area of the valve is given by

$$A_{\text{Eff}} = \frac{A_{\text{EO}} A_{\text{Ref}}}{A_{\text{Ref}} - A_{\text{EO}}}. \quad (6)$$

Valve opening and closure can be introduced by using a valve state $\sigma \in [0, 1]$ to switch between the effective orifice areas $A_{\text{EO,max}}$ and $A_{\text{EO,min}}$ in the fully open and closed states, respectively:

$$A_{\text{EO}}(t) = (A_{\text{EO,max}} - A_{\text{EO,min}}) \sigma(t) + A_{\text{EO,min}}. \quad (7)$$

By defining the minimum and maximum area ratios

$$M_{\max} = \frac{A_{\text{EO,max}}}{A_{\text{Ref}}} \quad \text{and} \quad M_{\min} = \frac{A_{\text{EO,min}}}{A_{\text{Ref}}}, \quad (8)$$

the effective area can be expressed as

$$A_{\text{Eff}}(t) = A_{\text{Ref}} \frac{s(t)}{1 - s(t)} \quad \text{with} \quad (9)$$

$$s(t) = (M_{\max} - M_{\min}) \sigma(t) + M_{\min}.$$

A smooth transition from a closed ($\sigma = 0$) to an open ($\sigma = 1$) valve can be accomplished by making the rate of opening proportional to $1 - \sigma$ and the rate of closure proportional to σ [6]:

$$\frac{d\sigma}{dt} = \begin{cases} K_o(1 - \sigma)\Delta p & \text{if } \Delta p > 0, \\ K_c\sigma\Delta p & \text{if } \Delta p \leq 0. \end{cases} \quad (10)$$

2.2. Model of Aortic Stenosis

Before we simulate aortic stenosis, we first have to establish suitable values for the parameters in Eqs. (5) to (10)

for healthy valves (Table 2). The exact size of the reference area A_{ref} is of little importance compared to the maximum area ratio. Therefore, we choose the reference area for the semilunar valves based on the diameter of the proximal pulmonary artery and aorta (typically around 3 cm) and assume a maximum area ratio close to 1. Atrioventricular (AV) valves typically have a larger diameter (4 cm to 5 cm). However, the annulus diameter is smaller, which is why we set the maximum area ratio to 70%. The proportionality constants K_o and K_c are responsible for how fast the valves open and close. We choose the values such that it takes a few milliseconds to open the valves and realistic reverse flows occur during valve closure.

Table 2. Parameters used for healthy heart valves.

Param.	Semilunar valves	Atrioventricular valves
A_{ref}	7 cm ²	15 cm ²
M_{max}	0.95	0.7
M_{min}	0.001	0.001
K_o	10 mmHg ⁻¹ s ⁻¹	20 mmHg ⁻¹ s ⁻¹
K_c	6 mmHg ⁻¹ s ⁻¹	6 mmHg ⁻¹ s ⁻¹

To simulate aortic stenosis, the maximum area ratio of the aortic valve is reduced to $M_{\text{max}} = 0.1$. This results in an effective orifice area of 0.7 cm², which is classified as severe stenosis ($A_{\text{EO,max}} < 1 \text{ cm}^2$) and should result in a mean pressure gradient $> 50 \text{ mmHg}$ according to AHA guidelines [10].

3. Results

To study the influence of the closing rate coefficient K_c , the healthy scenario was also simulated with $2 \cdot K_c$ and $3 \cdot K_c$. The resulting valve states, flows, and pressures on either side of the aortic and mitral valve are shown in Figure 2. In the case of the aortic valve, increasing K_c leads to a decrease in backflow and consequently the size of the dirotic notch. By further increasing K_c , the flow and pressure waveforms would slowly converge towards the results with diode-like valves. For the mitral valve however, increasing K_c has larger consequences. After early filling ("E" wave), LA and LV pressures are within 2 mmHg to 4 mmHg. If the mitral valve closes more rapidly, it may reopen again during diastole which results in LA and LV pressures to oscillate around each other resulting in mid-diastolic flow. In some cases, this flow can be observed as a distinct "L" wave. Compared to the diode-like valve, a delay between LA and LV pressure can be observed during phases in which the mitral valve is open. When inertia of the blood is neglected (Fig. 2, no dynamics), LA and LV pressure waveforms run synchronously.

The pressures and aortic flow of the simulation with severe aortic stenosis are shown in Fig. 3. Compared to

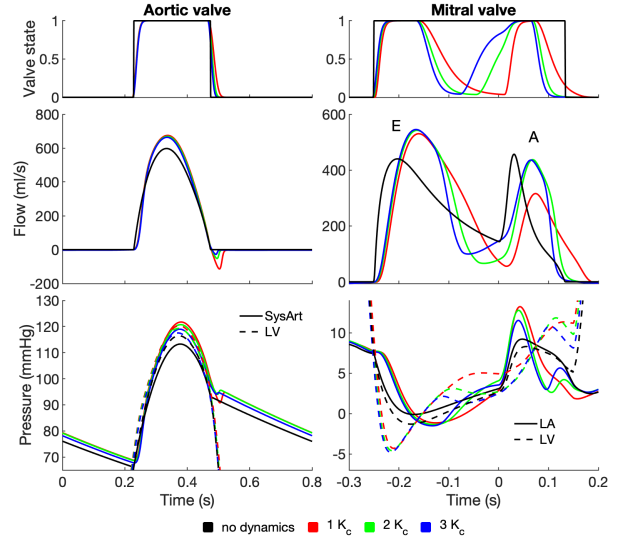


Figure 2. Influence of the closing rate coefficient K_c on the valve state, flow, and pressure of the aortic and mitral valve. The simulation with no dynamics refers to valves modeled as diodes as in [1]. Time for the mitral valve data was shifted by 300 ms for better visibility.

the simulation with a healthy valve, the stenosis causes the systolic aortic pressure to decrease by 30.5 mmHg, whereas the larger contribution to the pressure gradient ($\Delta p = 77.2 \text{ mmHg}$) is due to an increase in left ventricular (LV) pressure. Furthermore, the flow profile through the aortic valve becomes more rectangular shaped and stroke volume (SV) is reduced by 20.8 mL. An increase in mean left atrial pressure of 5.2 mmHg can be observed as well.

4. Discussion

In this study, we extended our previous model of the human circulatory system with a model of heart valve dynamics derived from Bernoulli's equation with smooth opening and closing of the valves. We demonstrated the validity of the model by simulating healthy valves as well as a severely stenotic aortic valve. In both cases, the model was able to faithfully reproduce all major features of typical pressure, flow, and volume measurements. Using an inertia in the description of the AV valves in particular leads to a more correct temporal relationship between atrial and ventricular pressures. During the early filling phase, ventricular pressures drop more than what is typically seen in measurements. The reason for this might be that the inertia (L in Eq.(5)) for the AV valves is too large, since the original formulation by Garcia et al. [5] was intended for the aortic valve only. Nevertheless, using the model for the AV valves led to a more accurate representation of E and A flow waveforms and thus better E/A-ratio, which is an

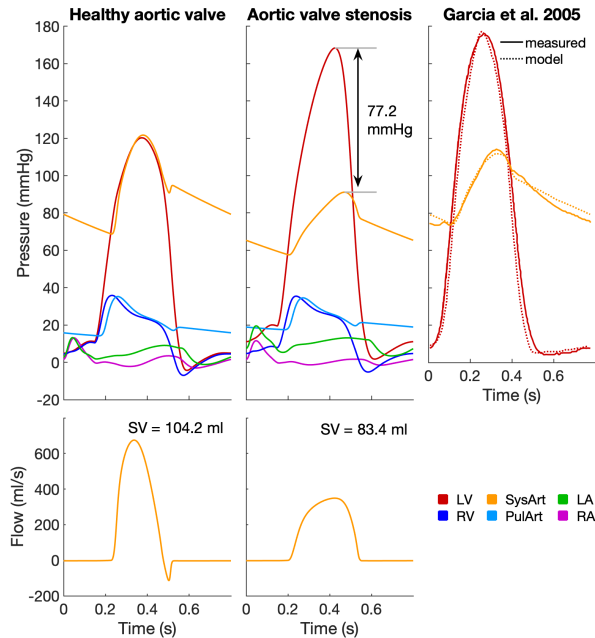


Figure 3. Results of simulations with a healthy aortic valve and severe aortic valve stenosis compared to measurements of a patient with severe aortic valve stenosis reproduced from [11].

indicator for LV diastolic function.

In the simulation of severe aortic stenosis, we observe results similar to measurements in patients pre aortic valve replacement surgery [11]. Due to the high outflow resistance of the stenotic aortic valve, LV emptying is impaired which results in a large pressure gradient across the valve. As a consequence peak systolic pressure within the left ventricle is increased. Additionally, this leads to increased wall stress (afterload), decreased SV, and increased end-systolic volume. The resulting pressure gradient of 77.2 mmHg matches the clinical classification of severe aortic stenosis.

In conclusion, we were able to improve the physiological fidelity of the 3D-0D whole heart model in terms of volumes, pressures, and flows by replacing the diode-like valve model with a model of heart valve dynamics based on Bernoulli's principle with smooth opening and closing of the valves. As demonstrated with the simulation of severe aortic stenosis, the model can be easily adapted to reflect common heart valve diseases such as stenosis or regurgitation.

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