Reconstructing Cardiac Voltage Using Data Assimilation: Effects of Observation Distribution

Shoale Badr[1], Flavio H Fenton[2], Elizabeth M Cherry[1]

[1]School of Computational Science and Engineering, Georgia Institute of Technology, Atlanta, GA, USA
[2]School of Physics, Georgia Institute of Technology, Atlanta, GA, USA

Abstract

Both experimental and clinical cardiac electrophysiology data often are recorded with limited spatial resolution or inconsistent spatial distribution. Rather than interpolating such data, which neglects constraints imposed by dynamics, data assimilation can be used to mitigate the effects of low-resolution data by providing estimates of voltage in unmapped areas. Here, we investigate the effects of different spatial distributions of observation data on cardiac voltage reconstructions over time using data assimilation. We used an ensemble Kalman filter method to reconstruct complete, uniform voltage data from observational data with different spatial distributions. Stable spiral-wave and sustained spiral-wave breakup cases from the modified Mitchell-Schaeffer model were analyzed. Coarse uniform observations and observations restricted to half the domain were considered. Stable spiral wave dynamics could be reconstructed well, with some lag at the coarsest resolutions and for half-domain observations far from the spiral core. Breakup cases were more difficult to match quantitatively; scenarios with fewer spirals or greater observation coverage of the most intense breakup regions could be reconstructed more accurately. Overall, we found that accurate voltage reconstructions could be generated for stable waves and breakup cases using data assimilation, provided that observations include regions driving breakup.

1. Introduction

Although accurate estimates of complex spatiotemporal cardiac electrophysiology states may facilitate several avenues of research, including customized defibrillation and ablation planning, accurate estimation of voltages in tissue remains a difficult challenge. One estimation option, simulation of tissue dynamics using cardiac electrophysiology models, has high spatial resolution and may represent many variables of interest, but model predictions are an approximation of real-world data. In contrast, cardiac electrophysiology recordings are often sparse in space or time and may not include all variables of interest. Although interpolation can be used to increase time or space resolution, this procedure ignores constraints imposed by dynamics and cannot predict other variables that may not be observed. In addition, interpolation may have difficulty in cases where portions of the domain may not be represented, such as in regions that are clinically inaccessible with mapping catheters or, for optical mapping, when tissue shape is irregular or dye is distributed unevenly.

Data assimilation is a commonly used estimation procedure that aims to merge the best aspects of numerical model predictions and real-world observational data. In previous work [1–4], we used the Local Ensemble Transform Kalman Filter [5] combined with the Fenton-Karma numerical prediction model [6] to reconstruct states in one and three spatial dimensions. Although we studied the roles of model error and various algorithmic parameters, our consideration of the impact of the spatial distribution of observations was limited to a comparison of two different uniform spatial resolutions in one dimension and to the use of spatially uniform vs. surface-only observations in three dimensions [1]. In the present work, we consider how the spatial distribution of observations affects the quality of state reconstruction by varying the spatial resolution of observations more widely and by utilizing spatially nonuniform observations within a two-dimensional domain.

2. Methods

2.1. Model

Our data-assimilation method used the two-variable modified Mitchell-Schaeffer model [7], which eliminates unwanted pacemaker behavior present in the original model. The model equations were integrated numerically using forward Euler with a time step of $0.25\,\mathrm{ms}$, spatial step of $0.05\,\mathrm{cm}$, and diffusion coefficient of $0.001\,\mathrm{cm}^2/\mathrm{ms}$. The domain size was $200{\times}200$, corre-

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sponding to a physical size of $10\,\mathrm{cm}\times 10\,\mathrm{cm}$.

Three parameter sets were used. Parameter set 1, which led to stable spiral waves following a meandering linear core, consisted of $\tau_{in} = 0.35\,\mathrm{ms}$, $\tau_{out} = 0.6\,\mathrm{ms}$, $\tau_{open} = 120\,\mathrm{ms}$, $\tau_{close} = 150\,\mathrm{ms}$, and $v_{gate} = 0.13$ For parameter set 2, which yielded spatially discordant alternans with breakup developing far from the spiral core, only three values were set differently: $\tau_{in} = 0.3\,\mathrm{ms}$, $\tau_{open} = 150\,\mathrm{ms}$, and $\tau_{close} = 170\,\mathrm{ms}$. Parameter set 3 produced sustained spiral wave breakup and used the same values as parameter set 1 except for $\tau_{in} = 0.3\,\mathrm{ms}$. Spiral waves in all cases were initiated using a cross-stimulation protocol, and a spin-up period was utilized to bypass transient behavior associated with initiation.

2.2. Datasets

Because our focus was on the effects of observation distributions on state reconstruction accuracy, we used voltage observations derived from the same model (no model error). Random Gaussian erorr with a mean of 0 and standard deviation of 0.1 was added to create the observations. Two types of observation distributions were considered: uniform but coarse observations corresponding to grids of 100×100 ($0.1\,\mathrm{cm}$ spacing), 50×50 ($0.2\,\mathrm{cm}$ spacing), and 25×25 ($0.4\,\mathrm{cm}$ spacing), and "half-domain" observations consisting of all voltage values from either the right half or the top half of the domain.

2.3. Data Assimilation

Briefly, data assimilation is an approach for generating state reconstructions from sparse and noisy observations by integrating predictions generated by a numerical model. For high-dimensional systems, such as our two-dimensional tissue simulations, efficiency is improved through the use of an ensemble of system states to characterize uncertainty. For this work, we adapted the Parallel Data Assimilation Framework (PDAF) [8] to support a two-dimensional implementation of cardiac tissue [9]. PDAF includes a variety of data-assimilation approaches; we used the Error Subspace Transform Kalman Filter (ESTKF) [10], which was designed to increase efficiency and performs computations within the error subspace described by the ensemble. The ensemble size was fixed at 10 members, observations were assimilated every ten time steps ($2.5\,\mathrm{ms}$), and runs lasted $4\,\mathrm{s}$. Initial ensemble members were generated from spinup states as in [3].

3. Results

Figure 1 shows voltage reconstructions from uniform coarse observations at two time points for a meandering linear core case without breakup. Good agreement is observed throughout the simulations. When the coarsest observations are used, a small degree of lag appears toward the later times, slightly distorting the spiral tip shape.

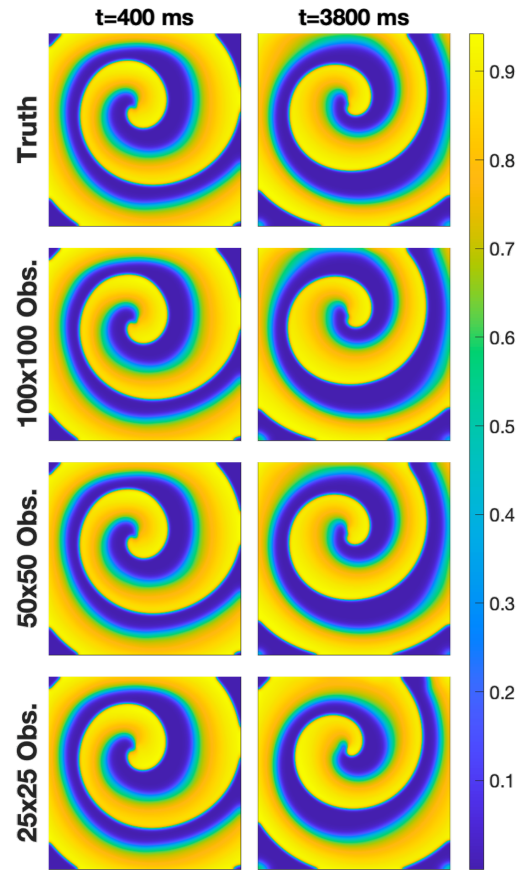

Figure 1. Truth state and voltage reconstructions for the linear core case (parameter set 1) using observations of varying coarseness.

When discordant alternans occurs, leading to breakup far from the core over time, the voltage estimates show greater error, as shown in Figure 2. Differences from the slight slowing of the wavefronts when the observations are coarsened are amplified in the setting of discordant alternans, leading to larger differences between the voltage reconstructions and the truth. While the estimates are generally accurate while the spiral wave is stable, once breakup begins, the fine "fibrillatory" dynamics is difficult to reproduce with quantitative accuracy. Nevertheless, even the coarsest set of observations yields an estimate that qualitatively matches the stable spiral wave with breakup far from the core. Figure 3 shows that that root mean square error (RMSE) remains consistent over the entire simulation; the RMSE oscillations correspond to the alternans period, which is roughly twice the rotation period.

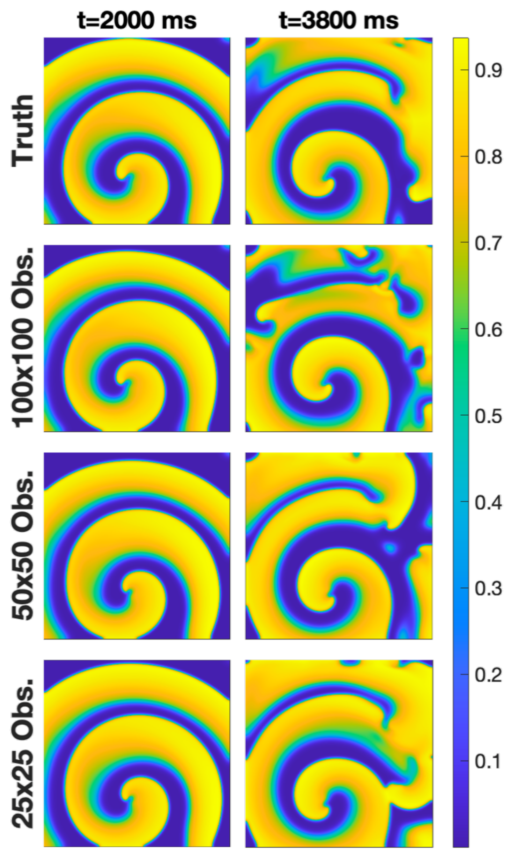

Figure 2. Truth state and voltage reconstructions for the discordant alternans case (parameter set 2) using observations of varying coarseness.

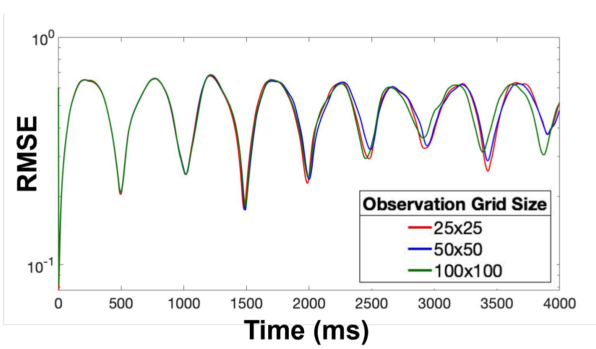

Figure 3. Root mean square error (RMSE) as a function of time for the discordant alternans case (parameter set 2) using observations of varying coarseness.

For the scenario with sustained spiral breakup, producing accurate estimates is more challenging, as can be seen in Figure 4. Early in the simulation, the dynamics include sharp spiral wave turns leading to the tip stalling and reforming. By 1 s, differences along the wave back near the core can be seen in the estimates except for the finest resolution. Spiral breakup begins soon after, characterized in this case by many short-lived waves with extremely small wavelengths. Data assimilation has difficulty recovering the details of these fine-scale patterns, as can be seen by the differences between the truth and voltage estimates after 2 s. Even the finest observations considered cannot reproduce the specific details of the truth state when it is characterized by sustained spatiotemporal chaos.

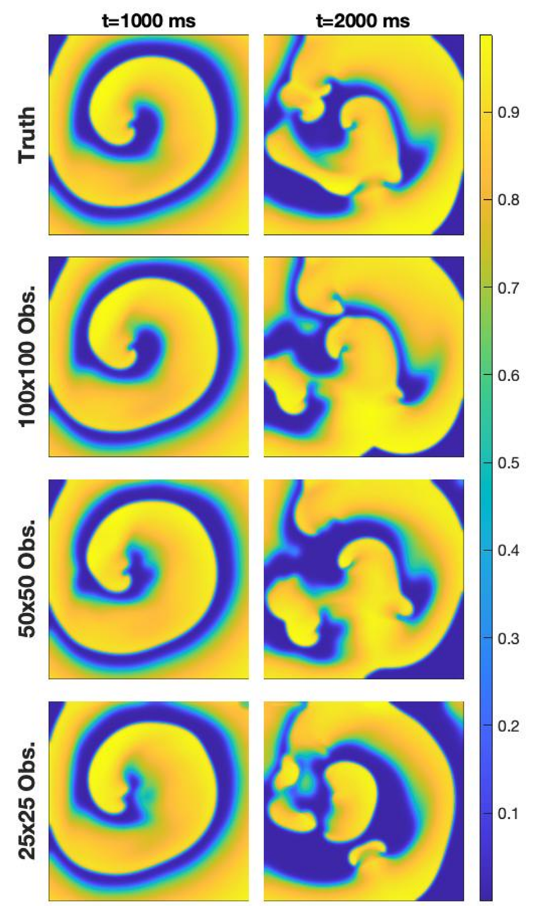

Figure 4. Truth state and voltage reconstructions for the breakup case (parameter set 3) using observations of varying coarseness.

In the cases considered so far, the observations tested were uniform throughout the grid, although at different spacings. As an additional scenario, we use observations restricted to one half of the domain for the discordant alternans dynamics of Figure 2. As shown in Figure 5, such a case can produce surprisingly good results. When observations are included on the right half of the domain, the estimates closely match the truth throughout the 4 s simulated, with only small discrepancies along the top and right edges. In contrast, when observations are restricted to the top half of the domain, the results are considerably worse; even the stable spiral wave is no longer captured properly, and discrepancies are visible even after 2 s, when breakup has not yet occurred.

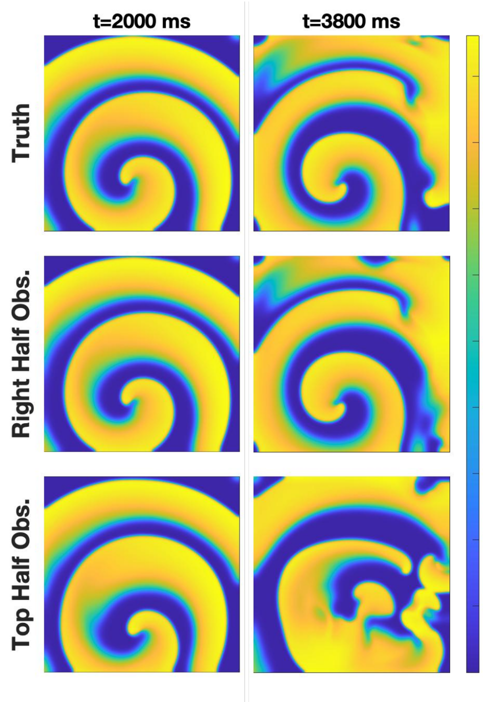

Figure 5. Truth state and voltage reconstructions for the discordant alternans case (parameter set 2) using observations in the right half and top half of the domain.

4. Conclusion

In this paper, we have considered the problem of state estimation of cardiac voltage for different two-dimensional dynamical regimes. We used an ensemble Kalman filter method with observations at different spatial distributions, including three uniform observation sets of varying coarseness and two sets of observations limited to half of the domain. In general, we found that data assimilation could produce quantitatively accurate voltage estimates when breakup did not occur. In breakup cases, coarse observation grids did not perform as accurately, but still produced qualitatively similar voltage estimates. Observations limited to half of the domain produced estimates with less predictable reliability, likely depending on the proximity of the the region with observations to the spiral wave core and the areas with the most severe breakup.

We note that we have considered only synthetic observations from a single model and a single assimilation interval; other choices could lead to different results. In addition, we have used only one type of data assimilation (ESTKF); other approaches may be more beneficial, including use of localization [3]. Future work is needed to study how observation spatial coverage impacts voltage estimates using experimental or clinical data.

Acknowledgments

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Address for correspondence:

Elizabeth M. Cherry
School of Computational Science and Engineering
Georgia Institute of Technology
756 West Peachtree Street Northwest
Atlanta GA 30332-4017
elizabeth.cherry@gatech.edu